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TRIGONOMETRIC SERIES FOR THE COODINATES OF THE OBJECTS IN THE SOLAR SYSTEM II THE OUTER PLANETS

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Abstract

Trigonometric series for approximate positions of some objects in the solar system have been developed by a nonlinear method of least squares. These give heliocentric and geocentric positions of the five outer planets. They are valid for the years 1950 to 2060 with a precision of 1″.

Key Words : method of least squares—trigonometric series—coordinates of celestial objects—multi-periodic phenomena

1. Introduction

It has been the desire of navigators and astronomers to know the positions of celestial objects using a simple procedure. As the recent diffusion of programmable calculators and personal computers have offered hardware for this purpose, software is needed to express the coordinates of celestial objects accurately and compactly.

Recently, Van Flandern and Pulkkinen (1979) have computed low precision formulae for the positions of the Sun, Moon, and the major planets. Their formulae have a precision of 1′ except for the case of Pluto (15′) and are valid within 300 years of the present. The medium-precision formulae for the Sun, Moon, and the inner planets have been obtained by Kubo (1980). His formulae have a precision of 0.1 through 60 years centered at the year 2000. Both of them were derived from analytical formulae using a certain series-processing program. Such a method is not, however, applicable to the case of the outer planets with a high precision. The present ephemerides for them are computed by a numerical integration so that there exists no analytical expression for the result. A new method valid in this case has been developed on the principle of least squares by the author (1981). The present paper gives high precision (1″) formulae for the positions of the outer planets which were obtained by this method.

2. Method

At first, the tables for the heliocentric polar coordinates and apparent geocentric rectangular coordinates of the outer planets for the years 1950 to 2060 are numerically obtained and stored in magnetic tapes. The number of data is 1013 for each coordinate. The time

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interval of them are 40 days. The sources of calculation are as follows. The heliocentric positions of the outer planets are obtained from the work of Eckert *et al.* (1951). The corrections by the inner planets are neglected because they are less than 0".1. The geocentric positions of the Sun are calculated by the formulae of Newcomb (1895). The precession and nutation are obtained from the formulae of Newcomb (1897) and Woolard (1953). Then the apparent positions are computed from all the sources above including aberration.

Next the approximation formulae are derived from the stored data using a nonlinear harmonic analyzer COMPAL. COMPAL is a FORTRAN program and has an ability to obtain a compact approximation formula expressed as a finite Fourier series from sequential data. Roughly speaking it analyzes data as follows : it computes a power spectrum of data, chooses a set of probable frequencies, optimizes them, and subtracts the determined frequency components from data. This procedure is iterated while the number of components is increased until the desired precision is obtained (see Fukushima, 1981).

Note that all numerical values including the arguments of cosine function in the present formulae are determined numerically so as to approximate the result of numerical integration. Therefore it may be dangerous to find any meaning or relation from the obtained results.

3. Usage of the formulae

The formulae consist of two parts. One is the heliocentric longitudes, latitudes, and radius vectors R of the five outer planets refered to the ecliptic and mean equinox of date. These are shown in Tables 1 through 5. The other is the apparent geocentric equatorial rectangular coordinates X, Y, and Z refered to the true equator and equinox of date. These are shown in Tables 6 through 10. The apparent right ascension and declination are given by

$$R.A.=\tan^{-1}(Y/X),\ Dec.=\tan^{-1}(Z/\sqrt{X^2+Y^2}). \tag{1}$$

The independent variable T is time measured from J2000.0 in Julian ephemeris centuries, or

$$T=(\mathrm{JED}-2\ 451\ 545.0)/36\ 525, \tag{2}$$

where JED denotes the Julian ephemeris day number. Radius vectors and rectangular coordinates are measured in AU. Longitudes, latitudes and arguments of cosine function are measured in degrees. It is recommended in evaluating the formulae to make the absolute value of arguments less than 180° by subtracting or adding 360° multiplied by some integer in order not to loose an accuracy.

The formulae are valid in the period 1950 Feb. 8.0 ET to 2060 Dec. 7.0 ET, i.e.

$$2\ 433\ 320.5 \leqq \mathrm{JED} \leqq 2\ 473\ 800.5, \tag{3}$$

or

$$-0.498\ 959\ 62 < T < +0.609\ 322\ 38. \tag{4}$$

We note that one should use the the formulae within the period above. They will have errors growing rapidly outside the period of validity, as is usual with best approximating formulae.

The maximum of absolute error from the source is listed at the end of each formula. For the users who need lower-precision formulae, Figures 1 through 6 are prepared. It can be

shown that the sum of the final error and the absolute value of coefficient of truncated terms gives an upper bound for the absolute error of remaining terms. These figures show the upper bound for the relative error of our formulae as a function of the number of remaining terms. Note that the upper bounds are measured in a logarythmic scale. When one needs formulae with a certain precision, one can truncate the original series as long as the upper bound in the corresponding figure is less than the desired precision. For example, the 1′-precision formulae for heliocentric longitude, latitude, and radius vector of Jupiter are given by the first 14, 4, and 9 terms of Table 1, respectively.

Table 11 gives the values of the formulae and the obtained right ascensions and declinations at 1969 June 28.0 ET, when JED=2 440 400.5 and T=−0.305 119 78, which is same as the test date of Van Flandern and Pulkkinen. The deviations from the *Japanese Ephemeris* are also shown in parentheses.

4. Discussion

Here we show in Figure 7, the figure for the periodic part of longitude PLON of the formulae of Van Flandern and Pulkkinen corresponding to Figure 1. Other figures for their formulae show almost the same tendency. Though these figures show only an upper bound of error, they can give an estimate for the cost performance of approximation formulae, i.e. how high accuracy is achieved with given number of memories. It is clear that our formulae are superior to theirs except for the period of validity. Why dose such a discrepancy arise ? The answer may be as following. In the analytical formulation the arguments are calculated as a number of linear combinations of fundamental arguments. Strictly they are different each other. However some of them are practically same since there are many commensurabilities among the fundamental arguments. Our program has such a practical nature that it may combine the terms with such similar arguments into a few terms. This can be said also in the case of terms with very long periods so that their difference from a linear trend is practically negligible.

Our formulae approximate the present ephemerides with a precision of 1″. However, we note that recent observations show that there is a deviation between the observed positions and the predicted ones which amounts to about 2″ in the case of Jupiter, Saturn and Uranus, and about 8″ in the case of Neptune and Pluto (Adams and Scott 1964, 1965, 1967, 1969, Cohen et,al. 1967, Klock and Scott 1970, 1972, Gauss 1979). Therefore we cannot assure that our formulae give the real positions of the outer planets with a precision of 1″.

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Table 1 Jupiter : longitude, latitude and radius vector.

LONGITUDE

		°			°			°		
+	3036.08406	* T *	COS	(	0.	* T +	0.	)		
+	34.39356	*	COS	(	0.	* T +	0.	)		
+	5.54603	*	COS	(	3034.53346	* T +	289.68429	)		
+	0.17575	*	COS	(	6083.2578	* T +	309.5012	)		
+	5532	*	COS	(	3624.312	* T +	237.453	)		
+	4370	* T *	COS	(	6071.843	* T +	218.916	)		
+	3557	*	COS	(	596.267	* T +	191.685	)		
+	2274	*	COS	(	2405.806	* T +	135.412	)		
+	2120	*	COS	(	1822.117	* T +	73.665	)		
+	757	*	COS	(	9107.13	* T +	327.94	)		
+	699	*	COS	(	2920.69	* T +	198.68	)		
+	607	*	COS	(	4759.11	* T +	88.37	)		
+	501	*	COS	(	5449.93	* T +	206.94	)		
+	482	*	COS	(	4730.71	* T +	273.36	)		
+	397	*	COS	(	4206.81	* T +	294.77	)		
+	343	*	COS	(	6666.05	* T +	259.11	)		
+	335	*	COS	(	1284.47	* T +	244.10	)		
+	224	*	COS	(	1466.26	* T +	54.64	)		
+	94	*	COS	(	7251.8	* T +	221.3	)		
+	39	*	COS	(	12131.7	* T +	345.5	)		
+	35	*	COS	(	8473.4	* T +	241.5	)		
+	29	*	COS	(	9704.4	* T +	270.6	)		
+	28	* T *	COS	(	9087.8	* T +	197.2	)		
+	12	*	COS	(	7779.9	* T +	258.5	)		
+	10	*	COS	(	10902.8	* T +	174.0	)		
+	9	*	COS	(	10324	* T +	251	)		
+	3	*	COS	(	12691	* T +	195	)		
±	0.00019									

LATITUDE

		°			°			°		
+	1.30086	*	COS	(	3034.12633	* T +	203.91874	)		
+	6299	*	COS	(	0.	* T +	0.	)		
+	6295	*	COS	(	6068.687	* T +	223.544	)		
+	2141	* T *	COS	(	3034.269	* T +	309.356	)		
+	347	*	COS	(	9134.65	* T +	243.22	)		
+	185	* T *	COS	(	9115.63	* T +	152.84	)		
+	106	* T *	COS	(	6049.97	* T +	339.44	)		
+	72	*	COS	(	2424.7	* T +	113.1	)		
+	63	*	COS	(	6654.4	* T +	171.6	)		
+	51	* T *	COS	(	0.	* T +	180.	)		
+	0.00045	*	COS	(	3627.8	* T +	122.2	)		
+	38	*	COS	(	1225.1	* T +	214.6	)		
+	33	*	COS	(	4850.1	* T +	0.8	)		
+	31	*	COS	(	562.9	* T +	266.8	)		
+	28	*	COS	(	5432.9	* T +	75.7	)		
+	20	*	COS	(	12136.2	* T +	262.6	)		
+	11	*	COS	(	4231.5	* T +	267.9	)		
+	7	*	COS	(	9699	* T +	191	)		
+	6	*	COS	(	8494	* T +	132	)		
+	6	*	COS	(	799	* T +	181	)		
±	0.00016									

RADIUS VECTOR

	AU				°			°		
+	5.209105		*	COS	(	0.	* T +	0.	)	
+	0.251681		*	COS	(	3034.534	* T +	199.614	)	
+	6134		*	COS	(	6066.1	* T +	219.0	)	
+	2802		*	COS	(	3624.5	* T +	147.7	)	
+	880		*	COS	(	2406	* T +	46	)	
+	635		*	COS	(	1818	* T +	344	)	
+	612	* T	*	COS	(	2968	* T +	19	)	
+	309	* T	*	COS	(	6090	* T +	312	)	
+	306		*	COS	(	601	* T +	97	)	
+	299		*	COS	(	5444	* T +	124	)	
+	224		*	COS	(	4215	* T +	205	)	
+	212		*	COS	(	9113	* T +	233	)	
+	129		*	COS	(	6671	* T +	171	)	
+	69		*	COS	(	7258	* T +	125	)	
+	61		*	COS	(	1190	* T +	169	)	
+	61		*	COS	(	4840	* T +	334	)	
+	57	* T	*	COS	(	9479	* T +	276	)	
+	30	* T	*	COS	(	0	* T +	180	)	
+	17		*	COS	(	1517	* T +	288	)	
+	16		*	COS	(	8482	* T +	167	)	
+	12		*	COS	(	7830	* T +	175	)	
+	9		*	COS	(	12122	* T +	249	)	
+	9		*	COS	(	10896	* T +	84	)	
+	5		*	COS	(	10256	* T +	164	)	
+	5		*	COS	(	9560	* T +	138	)	
±	0.000020									

Table 2 Saturn : longitude, latitude and radius vector.

LONGITUDE

	°				°			°		
+	1224.04687	* T	*	COS (	0.	* T	+	0.	)	
+	49.97937		*	COS (	0.	* T	+	0.	)	
+	6.19608		*	COS (	1178.39096	* T	+	227.52458	)	
+	4.81254	* T	*	COS (	1200.76570	* T	+	317.43950	)	
+	0.44141	* T	*	COS (	2380.3750	* T	+	279.0711	)	
+	0.19720		*	COS (	2310.5997	* T	+	187.9907	)	
+	0.11441		*	COS (	597.5060	* T	+	9.0137	)	
+	3220	* T	*	COS (	3731.729	* T	+	8.114	)	
+	1138		*	COS (	3816.622	* T	+	105.909	)	
+	857		*	COS (	1884.95	* T	+	334.19	)	
+	215		*	COS (	3151.76	* T	+	22.82	)	
+	184		*	COS (	5435.18	* T	+	44.09	)	
+	94		*	COS (	4198.7	* T	+	97.3	)	
+	56		*	COS (	4729.7	* T	+	316.6	)	
+	54		*	COS (	7243.0	* T	+	29.7	)	
+	40		*	COS (	4516.9	* T	+	147.7	)	
+	29		*	COS (	6009.6	* T	+	94.9	)	
+	18		*	COS (	9058.0	* T	+	13.7	)	
+	11		*	COS (	7825.9	* T	+	74.1	)	
+	10		*	COS (	6055	* T	+	92	)	
+	9		*	COS (	6619	* T	+	69	)	
+	7		*	COS (	10868	* T	+	354	)	
+	5		*	COS (	9616	* T	+	68	)	
+	4		*	COS (	8461	* T	+	79	)	
±	0.00014									

LATITUDE

	°				°			°		
+	2.47972		*	COS (	1224.56984	* T	+	206.33867	)	
+	0.13425		*	COS (	2445.2128	* T	+	163.7721	)	
+	5938	* T	*	COS (	1223.005	* T	+	113.635	)	
+	4858		*	COS (	0.	* T	+	0.	)	
+	978	* T	*	COS (	514.08	* T	+	27.67	)	
+	812		*	COS (	3673.28	* T	+	121.34	)	
+	305		*	COS (	437.68	* T	+	296.69	)	
+	0.00277		*	COS (	1812.03	* T	+	303.12	)	
+	230	* T	*	COS (	2450.45	* T	+	251.99	)	
+	64	* T	*	COS (	3629.0	* T	+	28.8	)	
+	55		*	COS (	3035.3	* T	+	321.0	)	
+	39		*	COS (	4861.3	* T	+	64.0	)	
+	17		*	COS (	4953.0	* T	+	79.5	)	
±	0.00010									

RADIUS VECTOR

	AU				°			°		
+	9.554001		*	COS (	0.	* T	+	0.	)	
+	0.515185		*	COS (	1188.045	* T	+	137.576	)	
+	0.314172	* T	*	COS (	1205.098	* T	+	227.317	)	
+	14780		*	COS (	2442.07	* T	+	92.09	)	
+	8245		*	COS (	1814.6	* T	+	341.2	)	
+	5400		*	COS (	586.3	* T	+	277.3	)	
+	1556		*	COS (	3612.4	* T	+	347.2	)	
+	1238	* T	*	COS (	2600.0	* T	+	225.3	)	
+	555	* T	*	COS (	0	* T	+	180	)	
+	314		*	COS (	5440	* T	+	313	)	
+	215		*	COS (	3941	* T	+	32	)	
+	166		*	COS (	3228	* T	+	160	)	
+	136		*	COS (	4900	* T	+	358	)	
+	119		*	COS (	4379	* T	+	22	)	
+	96		*	COS (	7245	* T	+	298	)	
+	55		*	COS (	6017	* T	+	15	)	
+	32		*	COS (	9051	* T	+	281	)	
+	27		*	COS (	7826	* T	+	1	)	
+	14		*	COS (	6628	* T	+	20	)	
+	12		*	COS (	10878	* T	+	263	)	
+	9		*	COS (	9610	* T	+	343	)	
+	7		*	COS (	8336	* T	+	15	)	
+	4		*	COS (	12669	* T	+	244	)	
±	0.000037									

Table 3 Uranus : longitude, latitude and radius vector.

LONGITUDE

	°					°			°	
+	428.72880	* T *	COS	(		0.	* T +		0.	)
+	313.33676		* COS	(		0.	* T +		0.	)
+	5.35857		* COS	(		460.61987	* T +		48.85031	)
+	3.20671	* T *	COS	(		705.15539	* T +		114.02740	)
+	2.69325	* T *	COS	(		597.77389	* T +		317.76510	)
+	0.58964		* COS	(		919.0429	* T +		188.3245	)
+	0.12397		* COS	(		1065.1192	* T +		354.5935	)
+	1475		* COS	(		2608.702	* T +		351.028	)
+	90		* COS	(		1968.3	* T +		247.7	)
+	36		* COS	(		5647.4	* T +		10.4	)
+	17		* COS	(		2356.6	* T +		183.6	)
+	17		* COS	(		2873.2	* T +		321.9	)
+	15	* T *	COS	(		3798.6	* T +		313.4	)
+	14		* COS	(		3157.9	* T +		308.1	)

± 0.00015

LATITUDE

	°					°			°	
+	1.78488	* T *	COS	(		507.52281	* T +		188.32394	)
+	1.15483		* COS	(		419.91739	* T +		128.15303	)
+	0.67756		* COS	(		652.9504	* T +		273.6644	)
+	0.56518	* T *	COS	(		892.2869	* T +		354.9571	)
+	0.13490		* COS	(		998.0302	* T +		83.3517	)
+	2997		* COS	(		0.	* T +		180.	)
+	36	* T *	COS	(		1526.5	* T +		263.0	)
+	25		* COS	(		3030.9	* T +		194.2	)

± 0.00014

RADIUS VECTOR

	AU					°			°	
+	19.203034		* COS	(		0.	* T +		0.	)
+	0.905790		* COS	(		408.729	* T +		320.313	)
+	0.361949	* T *	COS	(		440.702	* T +		19.879	)
+	0.166685	* T *	COS	(		702.024	* T +		307.419	)
+	62710		* COS	(		799.95	* T +		67.99	)
+	42617	* T *	COS	(		0.	* T +		0.	)
+	4897		* COS	(		2613.7	* T +		80.4	)
+	656		* COS	(		1527	* T +		202	)
+	223		* COS	(		2120	* T +		321	)
+	205		* COS	(		3104	* T +		37	)
+	120		* COS	(		5652	* T +		100	)

± 0.000064

Table 4 Neptune : longitude, latitude and radius vector.

LONGITUDE

	°					°			°	
+	219.93503	* T *	COS	(		0.	* T +		0.	)
+	55.13323		* COS	(		0.	* T +		180.	)
+	0.97450		* COS	(		221.3904	* T +		167.7269	)
+	4403	* T *	COS	(		684.128	* T +		332.797	)
+	2928	* T *	COS	(		904.371	* T +		342.114	)
+	1344		* COS	(		986.281	* T +		50.826	)
+	945		* COS	(		2815.89	* T +		0.09	)
+	235		* COS	(		2266.50	* T +		309.35	)
+	225		* COS	(		2279.43	* T +		127.61	)
+	23		* COS	(		5851.6	* T +		19.2	)

± 0.00009

LATITUDE

	°					°			°	
+	1.76958		* COS	(		218.87906	* T +		83.11018	)
+	1725		* COS	(		0.	* T +		0.	)
+	1366		* COS	(		447.128	* T +		338.864	)
+	15		* COS	(		1107.1	* T +		224.7	)
+	15		* COS	(		2596.7	* T +		187.5	)
+	12		* COS	(		3035.0	* T +		243.9	)

± 0.00009

RADIUS VECTOR

	AU					°			°	
+	30.073033		* COS	(		0.	* T +		0.	)
+	0.260457		* COS	(		222.371	* T +		79.994	)
+	9784	* T *	COS	(		515.2	* T +		195.7	)
+	4944		* COS	(		2815.4	* T +		90.1	)
+	3364		* COS	(		524.0	* T +		308.1	)
+	2579		* COS	(		1025.1	* T +		104.0	)
+	120		* COS	(		5845	* T +		111	)

± 0.000133

Table 5 Pluto : longitude, latitude and radius vector.

LONGITUDE

	°				°			°			
+	241.82574		* COS (		0.	* T +		0.	)		
+	179.09519	* T	* COS (		0.	* T +		0.	)		
+	15.81087		* COS (		246.556453	* T +		298.348019	)		
+	1.18379		* COS (		551.34710	* T +		351.67676	)		
+	7886		* COS (		941.622	* T +		41.989	)		
+	861		* COS (		2836.46	* T +		60.35	)		
+	590		* COS (		1306.75	* T +		112.91	)		
+	145		* COS (		2488.14	* T +		19.01	)		
+	22		* COS (		5861.8	* T +		77.9	)		
+	13		* COS (		3288.8	* T +		293.0	)		
±	0.00012										

LATITUDE

	°				°			°			
+	17.04550		* COS (		172.554318	* T +		42.574982	)		
+	2.45310		* COS (		415.60630	* T +		66.15350	)		
+	2.30285		* COS (		0.	* T +		180.	)		
+	0.26775		* COS (		713.1227	* T +		105.0840	)		
+	1855		* COS (		1089.202	* T +		146.660	)		
+	119		* COS (		2658.22	* T +		293.06	)		
+	98		* COS (		3055.6	* T +		18.8	)		
+	90		* COS (		1532.6	* T +		213.7	)		
+	42		* COS (		2342.3	* T +		254.2	)		
±	0.00010										

RADIUS VECTOR

	AU				°			°			
+	38.662489		* COS (		0.	* T +		0.	)		
+	8.670489		* COS (		181.3383	* T +		198.4973	)		
+	0.333884		* COS (		475.963	* T +		228.717	)		
+	8426		* COS (		909.8	* T +		252.9	)		
+	7619	* T	* COS (		1425.9	* T +		31.0	)		
+	4902		* COS (		2831.6	* T +		149.4	)		
+	2543	* T	* COS (		2196.1	* T +		199.5	)		
+	1188		* COS (		1748.0	* T +		114.1	)		
+	390		* COS (		3188	* T +		15	)		
+	116		* COS (		5860	* T +		169	)		
±	0.000050										

Table 6 Jupiter : apparent geocentric equatorial rectangular coordinates.

X

	AU				°			°	
+	5.196873		* COS (		3035.7093	* T +		34.3855	)
+	0.999832		* COS (		36000.768	* T +		280.434	)
+	0.370874		* COS (		0.	* T +		180.	)
+	0.125614		* COS (		6070.696	* T +		54.005	)
+	34057	* T *	COS (		3035.79	* T +		124.02	)
+	8745	* T *	COS (		436.5	* T +		180.3	)
+	8357		* COS (		71999.7	* T +		277.9	)
+	4552		* COS (		9103.8	* T +		73.6	)
+	2202		* COS (		449.6	* T +		97.7	)
+	1526		* COS (		2456.9	* T +		295.8	)
+	1410		* COS (		3637.6	* T +		314.3	)
+	1125		* COS (		6664.2	* T +		1.4	)
+	1098		* COS (		1280.2	* T +		51.7	)
+	795	* T *	COS (		5871	* T +		177	)
+	770		* COS (		4824	* T +		209	)
+	608		* COS (		5366	* T +		259	)
+	601		* COS (		908	* T +		62	)
+	196		* COS (		12139	* T +		93	)
+	168		* COS (		1576	* T +		208	)
+	164		* COS (		4430	* T +		56	)
+	136	* T *	COS (		9121	* T +		172	)
+	114		* COS (		1906	* T +		281	)
+	108		* COS (		107999	* T +		277	)
+	105		* COS (		5860	* T +		2	)
+	97		* COS (		9711	* T +		21	)
+	83		* COS (		8474	* T +		320	)
+	72		* COS (		3951	* T +		62	)
+	70		* COS (		7276	* T +		62	)
+	57	* T *	COS (		10061	* T +		98	)
+	54		* COS (		68966	* T +		105	)
+	42		* COS (		37937	* T +		155	)
+	42		* COS (		34066	* T +		226	)
+	38		* COS (		7888	* T +		209	)
+	31		* COS (		152542	* T +		357	)
+	30	* T *	COS (		71999	* T +		50	)
+	21		* COS (		29933	* T +		283	)
+	20	* T *	COS (		2443	* T +		188	)
+	17		* COS (		75039	* T +		55	)
+	16		* COS (		13496	* T +		27	)
+	15		* COS (		39025	* T +		54	)
+	10		* COS (		32981	* T +		320	)
+	10		* COS (		58517	* T +		8	)
+	8		* COS (		15171	* T +		112	)
+	8		* COS (		26897	* T +		293	)
+	7		* COS (		11500	* T +		1	)
+	6		* COS (		65928	* T +		59	)
+	6		* COS (		45028	* T +		74	)

± 0.000031

Y

	AU				°			°	
+	4.775585		* COS (		3035.8565	* T +		304.9512	)
+	0.917389		* COS (		36000.769	* T +		190.442	)
+	0.115475		* COS (		6070.741	* T +		324.606	)
+	70035		* COS (		0.	* T +		180.	)
+	18897	* T *	COS (		406.20	* T +		325.33	)
+	18613	* T *	COS (		3035.72	* T +		32.38	)
+	7667		* COS (		71999.8	* T +		187.9	)
+	4221		* COS (		9104.9	* T +		344.0	)
+	4093	* T *	COS (		1049.0	* T +		115.0	)
+	2255		* COS (		394.4	* T +		179.0	)
+	1966		* COS (		2427.8	* T +		201.6	)
+	1542		* COS (		1193.8	* T +		281.5	)
+	1195		* COS (		3596.8	* T +		226.8	)
+	1033		* COS (		6658.1	* T +		270.4	)
+	764		* COS (		4896	* T +		119	)
+	621		* COS (		5425	* T +		165	)
+	200	* T *	COS (		7625	* T +		233	)
+	186		* COS (		5148	* T +		285	)
+	179		* COS (		12141	* T +		4	)
+	177		* COS (		3943	* T +		231	)
+	155		* COS (		36033	* T +		97	)
+	149		* COS (		5847	* T +		267	)
+	99		* COS (		107999	* T +		187	)
+	83		* COS (		9673	* T +		291	)
+	83		* COS (		8469	* T +		231	)
+	55		* COS (		7086	* T +		318	)
+	47		* COS (		34064	* T +		136	)
+	44		* COS (		68968	* T +		342	)
+	30		* COS (		37939	* T +		65	)
+	28		* COS (		152543	* T +		267	)
+	22		* COS (		8027	* T +		111	)
+	18		* COS (		75032	* T +		325	)
+	18		* COS (		29931	* T +		51	)
+	16		* COS (		9374	* T +		99	)
+	15		* COS (		4246	* T +		71	)
+	14		* COS (		32963	* T +		226	)
+	14		* COS (		39027	* T +		326	)
+	13		* COS (		13445	* T +		294	)
+	11		* COS (		7544	* T +		71	)
+	10		* COS (		10254	* T +		274	)
+	9		* COS (		58514	* T +		278	)
+	9		* COS (		15170	* T +		20	)
+	6		* COS (		45027	* T +		344	)
+	6		* COS (		12761	* T +		308	)
+	6		* COS (		1877	* T +		211	)
+	5		* COS (		26949	* T +		221	)
+	5		* COS (		11510	* T +		261	)
+	4	* T *	COS (		35567	* T +		353	)

± 0.000029

Table 7 Saturn : apparent geocentric equatorial rectangular coordinates.

X

	AU				°			°	
+	9.515892		* COS (		1224.0425	* T +		49.9920	)
+	0.999833		* COS (		36000.768	* T +		280.409	)
+	0.257305		* COS (		2447.442	* T +		7.194	)
+	26965		* COS (		0.	* T +		0.	)
+	10553		* COS (		3667.90	* T +		324.49	)
+	8360		* COS (		71999.8	* T +		277.9	)
+	8220		* COS (		620.4	* T +		129.8	)
+	6914		* COS (		1817.3	* T +		146.9	)
+	5176		* COS (		233.3	* T +		278.8	)
+	4462		* COS (		3023.9	* T +		31.5	)
+	618		* COS (		2734	* T +		173	)
+	566		* COS (		4886	* T +		281	)
+	332	* T *	COS (		1904	* T +		52	)
+	264		* COS (		4236	* T +		239	)
+	160	* T *	COS (		5629	* T +		276	)
+	108		* COS (		107999	* T +		277	)
+	90		* COS (		4104	* T +		310	)
+	58		* COS (		70778	* T +		92	)
+	42		* COS (		37935	* T +		156	)
+	42		* COS (		34069	* T +		224	)
+	39		* COS (		5932	* T +		345	)
+	31		* COS (		152543	* T +		357	)
+	30		* COS (		73223	* T +		71	)
+	29		* COS (		7817	* T +		232	)
+	29		* COS (		6602	* T +		318	)
+	26		* COS (		37223	* T +		333	)
+	25		* COS (		34771	* T +		59	)
+	18		* COS (		9005	* T +		259	)
+	14		* COS (		13499	* T +		22	)
+	12		* COS (		8420	* T +		308	)
+	12		* COS (		33554	* T +		258	)
+	10		* COS (		9635	* T +		215	)
+	10		* COS (		58517	* T +		9	)
+	10		* COS (		7212	* T +		348	)
+	9		* COS (		68968	* T +		170	)
+	7		* COS (		29952	* T +		216	)
+	6		* COS (		26962	* T +		306	)
+	6		* COS (		45035	* T +		74	)
±	0.000045								

continued

Z

	AU				°			°	
+	2.049690		* COS (		3036.1022	* T +		301.4086	)
+	0.397731		* COS (		36000.737	* T +		190.432	)
+	49577		* COS (		6070.73	* T +		321.08	)
+	20352		* COS (		0.	* T +		180.	)
+	3324		* COS (		71999.8	* T +		187.9	)
+	3014	* T *	COS (		3016.7	* T +		120.3	)
+	1991		* COS (		134.1	* T +		102.7	)
+	1918		* COS (		599.1	* T +		222.4	)
+	1810		* COS (		9104.9	* T +		340.6	)
+	1287	* T *	COS (		1290.5	* T +		219.4	)
+	841		* COS (		2425	* T +		196	)
+	571		* COS (		1347	* T +		310	)
+	515		* COS (		3648	* T +		223	)
+	447		* COS (		6666	* T +		268	)
+	345	* T *	COS (		4914	* T +		351	)
+	345		* COS (		4922	* T +		136	)
+	304	* T *	COS (		36002	* T +		324	)
+	249		* COS (		5427	* T +		166	)
+	77		* COS (		12139	* T +		0	)
+	74		* COS (		5871	* T +		244	)
+	42		* COS (		107999	* T +		186	)
+	37		* COS (		37934	* T +		65	)
+	33		* COS (		9695	* T +		288	)
+	32		* COS (		8477	* T +		226	)
+	29		* COS (		4261	* T +		247	)
+	27		* COS (		7286	* T +		321	)
+	21		* COS (		7859	* T +		126	)
+	21		* COS (		68967	* T +		321	)
+	12		* COS (		152543	* T +		267	)
+	8		* COS (		29923	* T +		34	)
+	6		* COS (		13483	* T +		291	)
+	5		* COS (		39119	* T +		304	)
+	4		* COS (		33043	* T +		249	)
+	4		* COS (		58436	* T +		276	)
+	3		* COS (		15087	* T +		18	)
+	3		* COS (		72198	* T +		285	)
±	0.000024								

continued

Y

		AU			°		°	
+		8.808407		* COS (	1224.0387	* T +	320.9372	)
+		0.917398		* COS (	36000.772	* T +	190.409	)
+		0.691708		* COS (	0.	* T +	180.	)
+		0.237970		* COS (	2446.922	* T +	278.679	)
+		14146		* COS (	601.59	* T +	35.99	)
+		9739		* COS (	3666.1	* T +	234.6	)
+		7666		* COS (	71999.8	* T +	187.9	)
+		6146		* COS (	1827.9	* T +	60.8	)
+		3773		* COS (	3032.6	* T +	309.7	)
+		1900		* COS (	916.2	* T +	51.5	)
+		961	* T	* COS (	4138	* T +	110	)
+		441		* COS (	2238	* T +	11	)
+		440		* COS (	4902	* T +	183	)
+		294		* COS (	4030	* T +	350	)
+		169		* COS (	6027	* T +	331	)
+		99		* COS (	107999	* T +	187	)
+		51		* COS (	70773	* T +	303	)
+		49		* COS (	34074	* T +	136	)
+		44		* COS (	36110	* T +	66	)
+		42	* T	* COS (	33301	* T +	90	)
+		32		* COS (	73222	* T +	340	)
+		32		* COS (	5566	* T +	107	)
+		30		* COS (	7845	* T +	324	)
+		29		* COS (	37943	* T +	66	)
+		28		* COS (	152541	* T +	267	)
+		26		* COS (	9090	* T +	349	)
+		25		* COS (	37233	* T +	244	)
+		22		* COS (	6601	* T +	356	)
+		19		* COS (	34787	* T +	316	)
+		15		* COS (	13466	* T +	295	)
+		14		* COS (	35812	* T +	245	)
+		12		* COS (	7260	* T +	56	)
+		11		* COS (	9665	* T +	305	)
+		9		* COS (	5275	* T +	331	)
+		9		* COS (	68987	* T +	79	)
+		9		* COS (	58512	* T +	280	)
+		8		* COS (	33734	* T +	27	)
+		7		* COS (	26959	* T +	221	)
±		0.000050						

continued

Z

		AU			°		°	
+		3.654426		* COS (	1224.0851	* T +	314.4714	)
+		0.397731		* COS (	36000.769	* T +	190.773	)
+		0.286483		* COS (	0.	* T +	180.	)
+		98590		* COS (	2446.76	* T +	272.29	)
+		6352		* COS (	592.9	* T +	29.5	)
+		4025		* COS (	3672.6	* T +	228.3	)
+		3321		* COS (	71999.8	* T +	187.9	)
+		3026		* COS (	1369.9	* T +	49.4	)
+		2533		* COS (	35996.0	* T +	100.6	)
+		2392		* COS (	1862.9	* T +	52.9	)
+		1612		* COS (	3019.0	* T +	294.1	)
+		921	* T	* COS (	3173	* T +	316	)
+		517		* COS (	325	* T +	193	)
+		204		* COS (	4890	* T +	178	)
+		125		* COS (	4141	* T +	9	)
+		70		* COS (	6031	* T +	334	)
+		42		* COS (	107999	* T +	186	)
+		37		* COS (	37925	* T +	65	)
+		32		* COS (	70776	* T +	279	)
+		29		* COS (	4524	* T +	46	)
+		12		* COS (	152536	* T +	267	)
+		10		* COS (	37222	* T +	246	)
+		9		* COS (	5582	* T +	114	)
+		9		* COS (	9052	* T +	340	)
+		9		* COS (	34787	* T +	317	)
+		8		* COS (	7962	* T +	342	)
+		7		* COS (	13486	* T +	298	)
+		6		* COS (	7698	* T +	315	)
±		0.000047						

Table 8 Uranus : apparent geocentric equatorial rectangular coordinates.

X

	AU									
+	19.164292		* COS (	427.41034	* T +	313.23668	°	)		
+	1.341329		* COS (	0.	* T +	0.		)		
+	0.999834		* COS (	36000.769	* T +	280.355		)		
+	0.807731	* T	* COS (	421.560	* T +	45.270		)		
+	0.447773		* COS (	862.657	* T +	95.240		)		
+	15492		* COS (	1255.11	* T +	243.76		)		
+	8353		* COS (	71999.8	* T +	277.8		)		
+	5034		* COS (	3029.9	* T +	34.0		)		
+	4416	* T	* COS (	2065.4	* T +	275.0		)		
+	790		* COS (	1572	* T +	147		)		
+	278		* COS (	2675	* T +	198		)		
+	131	* T	* COS (	3481	* T +	355		)		
+	119		* COS (	6071	* T +	52		)		
+	107		* COS (	107999	* T +	276		)		
+	81		* COS (	71583	* T +	209		)		
+	61		* COS (	72422	* T +	335		)		
+	50		* COS (	35578	* T +	229		)		
+	41		* COS (	34061	* T +	225		)		
+	41		* COS (	37938	* T +	156		)		
+	41		* COS (	36423	* T +	155		)		
+	31		* COS (	152543	* T +	356		)		
+	28		* COS (	3553	* T +	80		)		
+	23		* COS (	4739	* T +	204		)		
+	17		* COS (	9012	* T +	240		)		
+	12		* COS (	35146	* T +	103		)		
+	11		* COS (	23225	* T +	98		)		
±	0.000094									

Y

	AU									
+	17.552054		* COS (	429.64339	* T +	223.58111	°	)		
+	0.917390		* COS (	36000.769	* T +	190.354		)		
+	0.398255		* COS (	896.175	* T +	5.692		)		
+	0.245942	* T	* COS (	914.113	* T +	270.330		)		
+	0.163874		* COS (	0.	* T +	180.		)		
+	24718	* T	* COS (	1519.08	* T +	51.59		)		
+	7662		* COS (	71999.7	* T +	187.8		)		
+	6717		* COS (	1592.1	* T +	150.8		)		
+	4481		* COS (	3055.0	* T +	304.7		)		
+	1322	* T	* COS (	3087.0	* T +	215.6		)		
+	558		* COS (	2285	* T +	98		)		
+	143	* T	* COS (	3853	* T +	199		)		
+	107		* COS (	6079	* T +	324		)		
+	99		* COS (	108000	* T +	187		)		
+	68		* COS (	72420	* T +	243		)		
+	66		* COS (	71589	* T +	24		)		
+	47		* COS (	36291	* T +	76		)		
+	47		* COS (	34072	* T +	134		)		
+	43	* T	* COS (	5537	* T +	59		)		
+	34		* COS (	35537	* T +	145		)		
+	31		* COS (	37961	* T +	66		)		
+	28		* COS (	152546	* T +	266		)		
+	24		* COS (	9034	* T +	342		)		
+	17		* COS (	4929	* T +	302		)		
+	12		* COS (	36652	* T +	55		)		
±	0.000088									

Z

	AU									
+	7.691820		* COS (	429.7127	* T +	221.5546	°	)		
+	0.397734		* COS (	36000.760	* T +	190.365		)		
+	0.188086		* COS (	868.697	* T +	2.712		)		
+	91580		* COS (	0.	* T +	180.		)		
+	12219		* COS (	1050.22	* T +	165.82		)		
+	10039	* T	* COS (	1556.57	* T +	49.62		)		
+	3322		* COS (	71999.7	* T +	187.8		)		
+	2110		* COS (	1658.7	* T +	143.5		)		
+	1947		* COS (	3038.8	* T +	301.1		)		
+	710		* COS (	2342	* T +	98		)		
+	100		* COS (	35888	* T +	96		)		
+	87	* T	* COS (	3975	* T +	220		)		
+	68	* T	* COS (	36385	* T +	153		)		
+	53		* COS (	71586	* T +	1		)		
+	42		* COS (	6073	* T +	319		)		
+	42		* COS (	108001	* T +	186		)		
+	38		* COS (	37926	* T +	65		)		
+	15		* COS (	3525	* T +	146		)		
±	0.000084									

Table 9 Neptune : apparent geocentric equatorial rectangular coordinates.

X

	AU			°		°	
+	30.085492		* COS (	219.77857	* T +	305.01054	)
+	0.999844		* COS (	36000.772	* T +	280.293	)
+	0.311795		* COS (	0.	* T +	180.	)
+	0.161623		* COS (	403.701	* T +	205.648	)
+	16536	* T * COS (		997.31	* T +	109.51	)
+	8467		* COS (	72002.5	* T +	277.3	)
+	4939		* COS (	3035.1	* T +	34.3	)
+	1457		* COS (	1146.8	* T +	112.8	)
+	1355		* COS (	1665.6	* T +	98.8	)
+	1322		* COS (	2116.5	* T +	176.7	)
+	611	* T * COS (		71933	* T +	154	)
+	119		* COS (	6073	* T +	52	)
+	110		* COS (	107999	* T +	276	)
+	51		* COS (	72299	* T +	342	)
+	41		* COS (	37947	* T +	156	)
+	40		* COS (	34064	* T +	225	)
+	31		* COS (	152544	* T +	356	)
±	0.000151						

Y

	AU			°		°	
+	27.823872		* COS (	219.95694	* T +	215.45252	)
+	0.917382		* COS (	36000.774	* T +	190.292	)
+	0.246372		* COS (	0.	* T +	180.	)
+	0.125691		* COS (	433.382	* T +	113.092	)
+	8073		* COS (	72004.4	* T +	184.9	)
+	4521		* COS (	3036.7	* T +	305.0	)
+	2529		* COS (	1231.7	* T +	321.9	)
+	1403		* COS (	1701.9	* T +	200.8	)
+	843		* COS (	2151	* T +	91	)
+	586		* COS (	72038	* T +	322	)
+	111		* COS (	6071	* T +	325	)
+	99		* COS (	108008	* T +	187	)
+	95	* T * COS (		72651	* T +	152	)
+	48		* COS (	2621	* T +	290	)
+	47		* COS (	34071	* T +	132	)
+	28		* COS (	152544	* T +	265	)
+	24		* COS (	9036	* T +	334	)
+	22		* COS (	3628	* T +	245	)
±	0.000151						

Z

	AU			°		°	
+	11.404508		* COS (	220.01995	* T +	211.67594	)
+	0.397740		* COS (	36000.729	* T +	190.292	)
+	94976		* COS (	0.	* T +	180.	)
+	46209		* COS (	443.20	* T +	109.43	)
+	3322		* COS (	71999.8	* T +	187.6	)
+	1930		* COS (	3037.2	* T +	301.5	)
+	1112		* COS (	2147.3	* T +	89.7	)
+	1057		* COS (	1217.0	* T +	314.9	)
+	377	* T * COS (		36011	* T +	314	)
+	139		* COS (	1725	* T +	8	)
+	78		* COS (	71781	* T +	2	)
+	48		* COS (	6073	* T +	321	)
+	41		* COS (	108008	* T +	186	)
±	0.000140						

Table 10 Pluto : apparent geocentric equatorial rectangular coordinates.

X

	AU								
+	38.257266		* COS (	149.51428	* T +	239.34175	°	)
+	10.160649		* COS (	0.	* T +	0.	°	)
+	3.892493		* COS (	352.9027	* T +	261.1047	°	)
+	0.999831		* COS (	36000.772	* T +	280.255	°	)
+	0.180553		* COS (	683.993	* T +	293.299	°	)
+	8127		* COS (	72004.1	* T +	277.2	°	)
+	6681	* T *	COS (	1880.2	* T +	80.7	°	)
+	5405		* COS (	1166.8	* T +	16.2	°	)
+	4934		* COS (	3036.2	* T +	34.4	°	)
+	4833		* COS (	2111.3	* T +	170.3	°	)
+	3053		* COS (	2145.6	* T +	11.7	°	)
+	401		* COS (	71886	* T +	289	°	)
+	362		* COS (	35787	* T +	344	°	)
+	346		* COS (	36218	* T +	33	°	)
+	120		* COS (	6069	* T +	52	°	)
+	111		* COS (	108000	* T +	276	°	)
+	41		* COS (	34058	* T +	224	°	)
+	41		* COS (	37939	* T +	156	°	)
±	0.000189							

Y

	AU								
+	32.360026		* COS (	129.18162	* T +	153.17524	°	)
+	11.621372		* COS (	0.	* T +	0.	°	)
+	10.378806		* COS (	256.64956	* T +	167.06002	°	)
+	0.917387		* COS (	36000.765	* T +	190.255	°	)
+	0.597494		* COS (	556.197	* T +	195.108	°	)
+	15014		* COS (	981.54	* T +	254.65	°	)
+	7691		* COS (	72002.1	* T +	187.4	°	)
+	5836	* T *	COS (	1957.7	* T +	132.5	°	)
+	4529		* COS (	3035.6	* T +	304.9	°	)
+	1648		* COS (	1562.7	* T +	311.2	°	)
+	323		* COS (	35781	* T +	263	°	)
+	284		* COS (	36229	* T +	301	°	)
+	110		* COS (	6065	* T +	325	°	)
+	100		* COS (	108008	* T +	186	°	)
+	73	* T *	COS (	23638	* T +	147	°	)
+	55		* COS (	71731	* T +	95	°	)
+	48		* COS (	2553	* T +	3	°	)
+	47		* COS (	34063	* T +	134	°	)
+	37		* COS (	72368	* T +	170	°	)
±	0.000174							

Z

	AU								
+	15.815501		* COS (	158.58605	* T +	109.93267	°	)
+	1.051941		* COS (	399.5038	* T +	135.1795	°	)
+	0.430833		* COS (	0.	* T +	0.	°	)
+	0.397751		* COS (	36000.767	* T +	190.250	°	)
+	40532		* COS (	748.40	* T +	169.01	°	)
+	3299		* COS (	72001.7	* T +	187.2	°	)
+	1943		* COS (	3035.9	* T +	301.5	°	)
+	1385		* COS (	1185.9	* T +	276.7	°	)
+	1198		* COS (	2139.5	* T +	30.9	°	)
+	192		* COS (	36187	* T +	297	°	)
+	164		* COS (	1827	* T +	109	°	)
+	103		* COS (	35762	* T +	261	°	)
+	48		* COS (	6068	* T +	321	°	)
+	41		* COS (	108006	* T +	185	°	)
±	0.000186							

Table 11 Values of series and coordinates on test date : 1969 June 28, 0^hET. The deviations from the *Japanese Ephemeris* are shown in parentheses

	JUPITER	SATURN	URANUS	NEPTUNE	PLUTO
LONGITUDE	188°34′02″.8 (+0″.1)	31°04′52″.0 (+0″.1)	183°13′35″.9 (−0″.2)	237°34′50″.3 (0)	174°25′42″.8 (−0″.2)
LATITUDE	+1°18′20″.0 (−0″.3)	−2°28′00″.1 (+0″.6)	+0°43′45″.3 (−0″.4)	+1°42′10″.2 (−0″.2)	+15°33′31″.1 (+0″.2)
RADIUS VECTOR	5.452 646 (+2)	9.262 811 (−5)	18.308 54 (+4)	30.323 38 (−4)	31.771 98 (+1)
X	−5.498 431	+7.818 230	−18.384 83	−16.355 44	−30.568 63
Y	+0.133 078	+5.468 890	−0.110 28	−22.904 41	−0.263 54
Z	+0.193 090	+1.937 260	+0.206 16	−8.951 10	+9.403 23
RIGHT ASCENSION	$11^h54^m27^s.25$ ($+0^s.01$)	$2^h19^m53^s.51$ ($+0^s.01$)	$12^h01^m22^s.48$ ($-0^s.01$)	$15^h37^m52^s.89$ ($-0^s.02$)	$11^h58^m01^s.45$ ($-0^s.01$)
DECLINATION	+2°00′38″.4 (+1″.0)	+11°28″.9 (0)	+0°38′32″.8 (−0″.2)	−17°38′34″.1 (+0″.1)	+17°05′52″.2 (0)

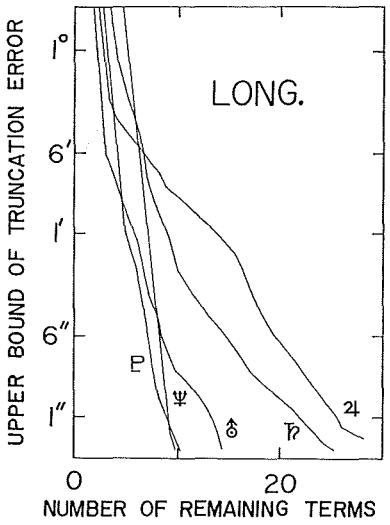

Figure 1 Upper bound of truncation error as a function of the number of remaining terms of the formulae for longitude. Note that the upper bound is measured in a logarythmic scale.

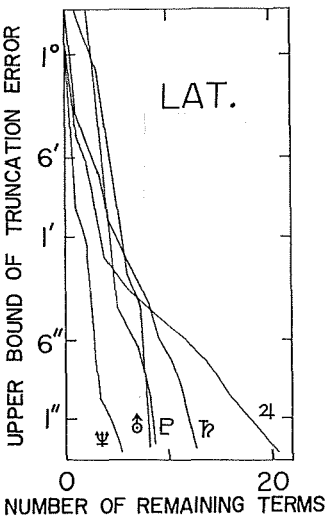

Figure 2 Same as Figure 1 but for latitude.

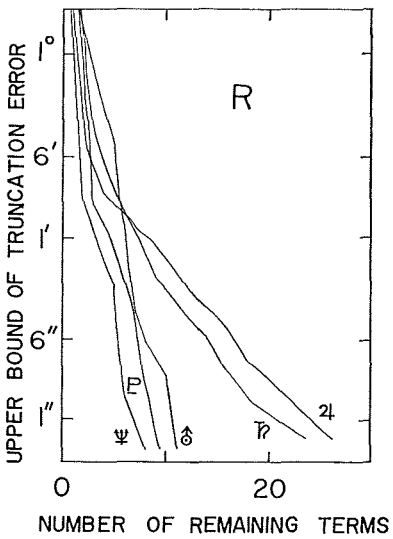

Figure 3 Same as Figure 1 but for radius vector. Note that the vertical axis is an upper bound for relative error.

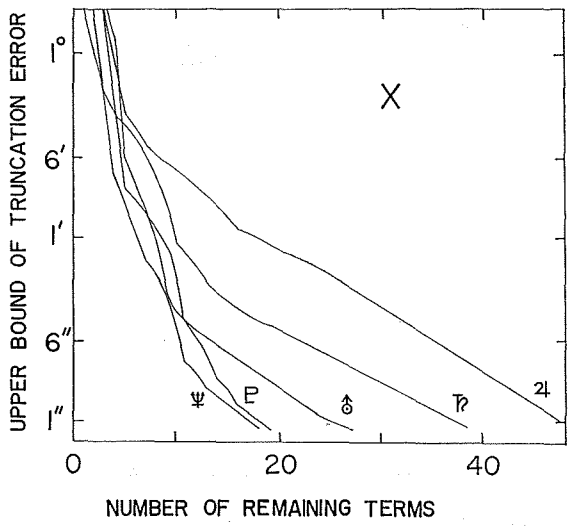

Figure 4 Same as Figure 3 but for X-coordinate of the apparent geocentric equatorial rectangular ones.

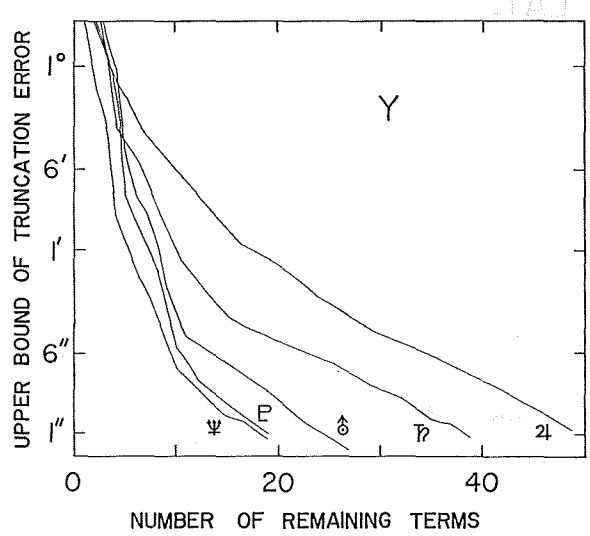

Figure 5 Same as Figure 3 but for Y-coordinate.

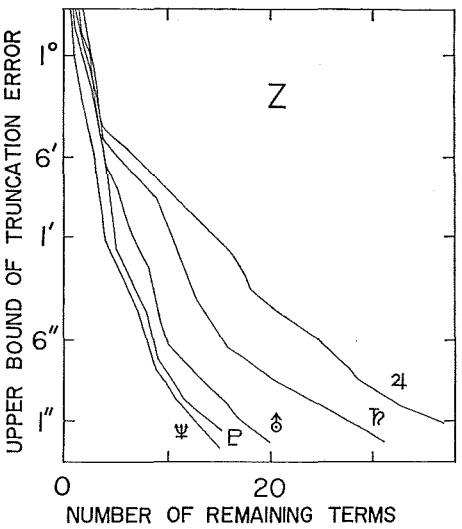

Figure 6 Same as Figure 3 but for Z-coordinate.

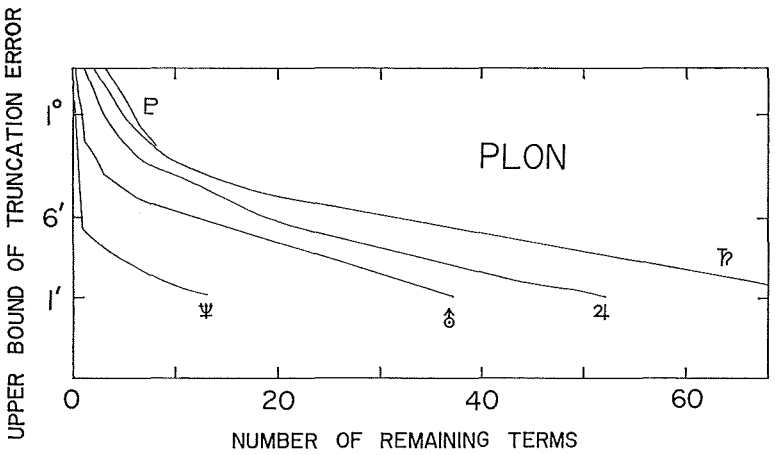

Figure 7 Same as Figure 1 but for the periodic part of longitude PLON of the formulae of Van Flandern and Pulkkinen (1979). The figures for other coordinates of their formulae show the same tendency. Note that the error for Pluto is greater than 15′.

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