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ALGORITHM FOR DETERMINATION OF A SATELLITE ORBIT AND GEODETIC PARAMETERS BY USING LASER RANGING DATA AND PRELIMINARY RESULTS OF ITS APPLICATION†

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Abstract

A satellite laser ranging system was installed at the Simosato Hydrographic Observatory and the satellite observation has been continued since March, 1982. To process the range data and to obtain satellite orbits and geodetic parameters, an orbital processor using numerical integration has been developed. The processor includes the terms of the non-spherical force due to the geopotential, lunisolar and planetary forces, radiation pressure, atmospheric drag and tidal effects of the solid earth and ocean. The algorithm and formation of the processor are described here.

The processor is applied to the range data obtained at the observatory and other laser sites in the world to determine the position of Simosato site in the global geocentric coordinate system. The coordinate of the intersecting point of azimuth and elevation axes of the laser ranging system at Simosato site is obtained on the basis of the LPM 81.12 coordinate system. The preliminary result is 33° 34′ 39″. 697N (latitude), 135° 56′ 13″. 156E (longitude) and 100.66 meters (height from the reference ellipsoid: A=6378 137.0m, 1/f=298.257). The comparison of the result with the geodetic coordinate surveyed in the Tokyo Datum derives the datum shift correction from the Tokyo Datum to the LPM 81.12 system as $\Delta U=-142.5$m, $\Delta V=+510.4$m and $\Delta W=+681.2$m. If the position of the origin of the Tokyo Datum is expressed in the LPM 81.12 system by using the datum shift correction, the values of the position is shifted by +11″.71 in latitude and −11″.83 in longitude on the basis of the values expressed in the Tokyo Datum and the shift amounts to 468 meters to the direction of 321 degrees in azimuth.

According to the results of the lunar laser ranging, an eastward rotation of the LPM 81.12 system of 0″.197 makes the same longitude for the reference point of the 2.7 meter telescope at the McDonald Observatory. If it is applied to the longitude of Simosato site, the datum shift correction changes to $\Delta U=-146.0$m, $\Delta V=+506.7$m and $\Delta W=+681.2$m. The new expression for the position of the origin of the Tokyo Datum obtained by using this datum shift correction on the basis of the lunar longitude system is given as 35° 39′ 29″. 217N (latitude), 139° 44′ 28″. 878E (longitude).

Key words: Orbital determination, Minimum Variance estimate, Satellite laser ranging, Tokyo Datum

1. Introduction

More than twenty fixed stations and two transportable stations for satellite laser ranging (SLR) are operating in the world at present (Smith 1983) and the range accuracies of these SLR

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systems are from a few centimeters to one meter (Pearlman 1983). The orbit of Lageos and other laser reflective satellites have been determined well. The root mean square (RMS) of the residuals to observed range minus calculated range for the Lageos orbit determined to estimate the earth rotation parameters reaches twenty centimeters level (Schutz 1983b). The Lageos orbits played the major part for determination of the pole motion of the earth by BIH in the MERIT Short Campaign (Tapley 1983) and will play an important role in the Main Campaign of the Project MERIT from September 1, 1983 to October 31, 1984.

On the other hand, the National Aeronautics and Space Administration (NASA) of the United States has been performed the Crustal Dynamics Project to measure crustal movements in the large area and to detect the continental plate motions by using the SLR and VLBI. As for the SLR in the project two transportable systems were developed (Silverberg 1980, Smith 1983) and the third transportable system will be deployed in 1984 (Dunn 1983). In Europe a joint team of West Germany and the Netherlands also has been making transportable systems (Wilson 1982).

In Japan the Tokyo Astronomical Observatory originated the SLR and gave much achievements (e.g. Kozai *et al.* 1973). The Hydrographic Department of Japan (JHD) installed a SLR system at the Simosato Hydrographic Observatory (SHO) (Sasaki *et al.* 1983). The operation started in March 1982 and the system has been running since then (Sasaki 1982b, Sasaki *et al.* 1984). The range accuracy of the system reached a few centimeters (Pearlman 1983). The launch of the Geodetic Satellite in early 1986 has also been prepared by the National Space Development Agency of Japan. The satellite has globular form with 2.15 meters of diameter. It is made of the synthetic resin and weighs 700 kilograms. The surface reflects the sunlight and the corner-cube-reflectors on the parts of the surface also reflect the laser beam to the ground. The launch orbit is circular with 1500 kilometers of altitude and 50 degrees of inclination (Funo 1983). A transportable SLR system will be made for the satellite project by JHD.

As for the orbital processing in such a background there are several processors, e.g. named UTOPIA (McMillan 1973), GEODYN (Martin *et al.* 1976) or KOSMOS (Murata 1978). However these processors are not convenient to treat the SLR data obtained at SHO for the author and staffs of JHD. And these processors did not have new astro-geodetic constants and systems. So the author made an orbital processor based on a linear estimation theory (Tapley 1973) to treat SLR data obtained at SHO and other sites and to analyze orbits, station coordinates and some geodetic parameters. An outline of the processor and the results of a set of the baseline determination were reported in a previous paper (Sasaki 1982a). The algorithm and formation of the processor and preliminary results for the coordinate of Simosato site by processing the SLR data obtained at SHO and other sites are presented in this paper.

2. Algorithm for estimation of the satellite orbit and geodetic parameters

(1) Basic relation

The equations of motion of a satellite around the earth in a non-rotating coordinate system are expressed by the first order differential equations as

$$\dot{r} = v \quad , \qquad \dot{v} = -\mu\frac{r}{r^3} + R(r, v, \alpha, t) \tag{1}$$

where r: position vector of the satellite, v: velocity vector, μ: geocentric constant of gravitation ($=GM$), R: perturbation acceleration which is a function of r, v, a set of model parameters α and time t. In the strict sense R is unknown. However if the unmodeled error can be ignored R can be expressed explicitly. If some geodetic constant parameters whose values should be estimated in the estimation procedure are denoted by a vector β, the equation of motion is

$$\dot{\beta}=0. \tag{2}$$

Equations (1) and (2) can be rewritten using an n-dimensional state vector X, which denotes the position and velocity of the satellite and the geodetic parameters to be estimated, as following equation:

$$\dot{X}=F(X,t), \qquad \textit{initially} \; X(t_0)=X_0. \tag{3}$$

Usually the state vector is related to observed values non-linearly. The observations are also influenced by random observation errors. An m-dimensional i-th observation vector, Y_i, observed at t_i is expressed with an error vector ε_i as

$$Y_i=G(X_i,t_i)+\varepsilon_i, \qquad i=1,2,\cdots\cdots,\ell, \tag{4}$$

where $G(X_i, t_i)$ is m-vector of a non-linear function relating the state and observation.

(2) Linearization

To estimate the state of a non-linear dynamical system the linearization is one of the most convenient method. In such a linearization method it is important to find a good approximation at first and to consider errors due to linearization assumption.

If the difference between $X(t)$ and a reference trajectory $X^*(t)$ is sufficiently small in the duration $t_0 < t < t_{max}$, equations (3) and (4) can be expanded around the reference trajectory as

$$\begin{aligned} \dot{X} &= \dot{X}^*+[\partial F/\partial X]^*(X-X^*)+\cdots\cdots, \\ Y_i &= G(X_i^*, \; t_i)+[\partial G/\partial X]_i^*(X_i-X_i^*)+\cdots\cdots+\varepsilon_i. \end{aligned} \tag{5}$$

If the terms of $(X-X^*)^2$ are neglected and the definitions

$$\begin{aligned} x(t)&=X(t)-X^*(t), \qquad & A(t)&=[\partial F/\partial X]^*, \\ y_i&= Y_i-G(X_i^*, \; t_i), \qquad & \tilde{H}_i(t_i)&=[\partial G/\partial X]_i^*, \end{aligned} \tag{6}$$

are used, equations (3) and (4) can be rewritten as

$$\dot{x}=A(t)x, \qquad \text{initially} \;\; x(t_0)=x_0, \;\; t_0<t<t_{max} \tag{7}$$

and

$$y_i=\tilde{H}_i(t_i)x_i+\varepsilon_i, \qquad i=1,2,\cdots\cdots,l \tag{8}$$

If a state transition matrix $\Phi(t_i, t_k)$ is introduced into the linear estimation theory the equation

$$x_i= \Phi_i(t_i, \; t_0)x_0 \tag{9}$$

is the solution of equation (7), and equation (8) becomes

$$y_i=\tilde{H}_i(t_i)\Phi(t_i, \; t_0)x_0+\varepsilon_i. \tag{10}$$

The state transition matrix satisfies the following relations (e.g. Liebelt 1967):

$$\begin{aligned} &\text{i)} \; \Phi(t_i, \; t_k)=\frac{\partial x(t_i)}{\partial x(t_k)} \qquad && \text{iv)} \; \Phi(t_i, \; t_k)=\Phi^{-1}(t_k, \; t_i), \\ &\text{ii)} \; \Phi(t_i, \; t_i)=I=\Phi(t_k, \; t_k), && \text{v)} \; \dot{\Phi}(t, \; t_k)=A(t)\Phi(t, \; t_k). \\ &\text{iii)} \; \Phi(t_i, \; t_k)=\Phi(t_i, \; t_j)\Phi(t_j, \; t_k), && \end{aligned} \tag{11}$$

If definitions

$$y=\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_l \end{bmatrix}, \quad H=\begin{bmatrix} \tilde{H}_1\Phi(t_1,\ t_0) \\ H_2\Phi(t_2,\ t_0) \\ \vdots \\ \tilde{H}_l\Phi(t_l,\ t_0) \end{bmatrix}, \quad \varepsilon=\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_l \end{bmatrix},$$

are used, all the observation equations can be expressed as

$$y=Hx_0+\varepsilon\,. \tag{12}$$

(3) Minimum variance estimate

To solve n-unknowns, x_0, and $(l \times m)$-unknowns, ε, from $(l \times m)$-observations, y, in equation (12) is the proposition of this problem. As the number of unknowns is greater than the number of observations, some constraint condition is necessary. In this estimation, to minimize the diagonal elements of the covariance matrix,

$$P=E[(\hat{x}-E[\hat{x}])(\hat{x}-E[\hat{x}])^T], \tag{13}$$

is adopted as the constraint condition, where E means to take expecting value and $\hat{x}$ denotes the solution of this statistic process. It is assumed that the observation error ε_i satisfies the a priori statistics as

$$E[\varepsilon_i]=0, \qquad E[\varepsilon_i\varepsilon_j^T]=R_i\delta_{ij}, \tag{14}$$

where δ_{ij} is the Kronecker delta and R_i is an element of a positive definite matrix, R.

As the solution of equation (12), the best linear unbiased minimum variance estimate, $\hat{x}_0$, of state x_0 is obtained by satisfying conditions described above, namely:

i) linearity expressed as $x = My$,

ii) unbias $E[\hat{x}] = x$,

iii) minimum variance $\partial Pii/\partial x = 0$.

The expression is given (e.g. Liebelt 1967) as

$$\hat{x}_0=(H^TR^{-1}H)^{-1}H^TR^{-1}y.$$

The best estimate $\hat{x}_0$ of x at $t = t_0$ obtained from the observations $y_1, y_2, \ldots.\ y_l$ at $t = t_1, t_2, \ldots.\ t_l$ is also expressed as

$$\hat{x}_0=E[x_0 \mid y_1,\ y_2,\ \cdots\cdots y_l]. \tag{15}$$

If $\bar{x}_k$ denotes a future prediction of x at $t = t_k\ (>t_l)$, $\bar{x}_k$ should be also obtained from the same observations $y_1, y_2, \ldots.\ y_l$ as written by

$$\overline{x_k}= E[x_k | y_1, y_2 \cdots\cdots y_l].$$

The relation of equation (9) follows that

$$E[x_k|y_1,y_2,\cdots\cdots y_l]=\Phi(t_k,t_0)E[x_0|y_1,y_2,\cdots\cdots y_l]\,.$$

and prediction $\bar{x}_k$ of x at $t = t_k$ is given by the equation,

$$\bar{x}_k=\Phi(t_k,t_0)\hat{x}_0. \tag{16}$$

The expression of the covariance matrix of x_0 is rewritten using equations (13), (12) and (14) as

$$\begin{aligned} P_0 &= E[(\hat{x}_0 - x_0)(\hat{x}_0 - x_0)^T] \\ &= E[((H^T R^{-1} H)^{-1} H^T R^{-1}(Hx_0 + \varepsilon) - x_0)((H^T R^{-1} H)^{-1} H^T R^{-1}(Hx_0 + \varepsilon) - x_0)^T] \\ &= ((H^T R^{-1} H)^{-1} H^T R^{-1}) E[\varepsilon \varepsilon^T] (H^T R^{-1} H)^{-1} H^T R^{-1})^T \\ &= (H^T R^{-1} H)^{-1}, \end{aligned} \tag{17}$$

namely

$$\hat{x}_0 = P_0 H^T R^{-1} y. \tag{18}$$

If a future prediction of the covariance matrix at $t = t_k$ is defined by

$$\bar{P}_k = E[(x_k - \bar{x}_k)(x_k - \bar{x}_k)^T | y_1, y_2, \cdots\cdots y_l], \tag{19}$$

this equation can be rewritten as followings

$$\begin{aligned} &= E[\Phi(t_k, t_0)(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T \Phi^T(t_k, t_0) | y_1, y_2, \cdots\cdots y_l] \\ &= \Phi(t_k,\ t_0) P_0 \Phi^T(t_k,\ t_0). \end{aligned} \tag{20}$$

It is considered to add a set of additional observation y_k at $t = t_k$ $(>t_l)$ to observations $y_1, y_2, \ldots y_l$ already obtained or to add additional observation to a priori estimate $\hat{x}_0$ and associated covariance matrix P_0 at $t = t_0$. The additional observation is expressed as

$$y_k = \tilde{H}_k x_k + \varepsilon_k. \tag{21}$$

In the case of addition of new observation to several sets of old observations, the new observation can be added into the matrix expression similarly as

$$\begin{bmatrix} y_1 \\ \vdots \\ y_l \\ \hdashline y_k \end{bmatrix} = \begin{bmatrix} \tilde{H}_1 \Phi(t_1,\ t_0) x_0 \\ \vdots \\ \tilde{H}_l \Phi(t_l,\ t_0) x_0 \\ \hdashline \tilde{H}_k \qquad x_k \end{bmatrix} + \begin{bmatrix} \varepsilon_i \\ \vdots \\ \varepsilon_l \\ \hdashline \varepsilon_k \end{bmatrix}$$

or

$$\begin{bmatrix} y \\ \hdashline y_k \end{bmatrix} = \begin{bmatrix} H\Phi(t_0,\ t_k) \\ \hdashline \tilde{H}_k \end{bmatrix} x_k + \begin{bmatrix} \varepsilon \\ \hdashline \varepsilon_k \end{bmatrix}. \tag{22}$$

If equation (22) is expressed as $z_k = H_k x_k + e_k$, the solution of this equation can be obtained similarly to equation (18):

$$\begin{aligned} \hat{x}_k &= (H_k^T R'^{-1} H_k)^{-1} H_k^T R'^{-1} z_k \\ &= \left\{ (\Phi^T(t_0,\ t_k) H^T \vdots \tilde{H}_k^T) \begin{pmatrix} R^{-1} & \vdots & 0 \\ \hdashline 0 & \vdots & R_k^{-1} \end{pmatrix} \begin{pmatrix} H\Phi(t_0,\ t_k) \\ \hdashline \tilde{H}_k \end{pmatrix} \right\}^{-1} (\Phi^T(t_0,\ t_k) H^T \vdots \tilde{H}_k^T) \begin{pmatrix} R^{-1} & \vdots & 0 \\ \hdashline 0 & \vdots & R_k^{-1} \end{pmatrix} \begin{pmatrix} y \\ \hdashline y_k \end{pmatrix} \\ &= (\tilde{H}_k^T R_k^{-1} \tilde{H}_k + \bar{P}_k^{-1})^{-1} (\tilde{H}_k^T R_k^{-1} y_k + \bar{P}_k^{-1} \bar{x}_0). \end{aligned} \tag{23}$$

In the case of addition of new observation to a priori estimate $\hat{x}_0$ and covariance matrix P_0, the relation $\bar{x}_k = x_k + \eta_k$ is applied into equation (22) instead of old observations and equation (12), where η_k is error of estimation for x_k and $E[\eta_k] = 0$, $E[\eta_k \eta_k^T] = \bar{P}_k$ as

$$\begin{bmatrix} \bar{x}_k \\ y_k \end{bmatrix} = \begin{bmatrix} I \\ \tilde{H}_k \end{bmatrix} x_k + \begin{bmatrix} \eta_k \\ \varepsilon_k \end{bmatrix}$$

and

$$R'^{-1} = \begin{bmatrix} \bar{P}_k & 0 \\ 0 & R_k^{-1} \end{bmatrix}.$$

The result of this case becomes the same expression as equation (23). The equation (23) gives the best estimate of x in the case to add new observation.

3. Dynamical models and their formulation

In this processor several dynamical models are used. The principal dynamical models are given in Table 1. Precise expressions will be given in the following sections.

(1) Time system

There are some time systems to express dynamical models conveniently. As the basic time system, invariable-, continuous- and observable time system should be adopted. So, TAI (International Atomic Time) is used for the basic time system of the processor. To express the independent variable of motion of the moon, the sun and planets, ET (Ephemeris Time) is used. UT1 is used for the parameter of time to denote the earth rotation and UTC (Universal Coordinated Time) is for observation time. The relation,

$$\mathrm{ET} - \mathrm{TAI} = 32.184^{\mathrm{S}},$$

(e.g. McMillan 1973) is used. For UT1R – TAI and UTC – TAI at each observation time the values are provided by e.g. BIH or USNO.

Table 1 Adopted system of dynamical models

Astronomical constants	IAU1976 System (1976)
Precession	Lieske, J.H. *et al.* (1977)
Nutation	Wahr, J.M. (1979)
Pole position	CIO
Definition of UT	Aoki, S *et al.* (1982)
Geopotential	GEM L2 (1983)
Earth model	1066A(Gilbert.F. and A.M. Dziewonski, 1975)
Solid earth tide and its site displacement	Shen, P.Y. and L. Mansinha (1976), Sasao, T. *et al.* (1977) and Wahr, J.M. (1979)
Ocean tide and its loading	Schwiderski, E.W. (1978)
Site displacement	Goad, C.C. (1980), Sasao, T. and I. Kikuchi (1982)
Tidal variation in UT1	Yoder, C.F. *et al.* (1981)
Air drag	exponential atmosphere
Radiation pressure	MERIT Standards (1983)
Satellite constants	ibid.
Luni-, Solar- and Planetary position	Japanese Ephemeris (1980)

(2) **Coordinate system**

A non-rotating coordinate system to the inertial space should be used to express the equation of motion of the satellite simply. As the basic non-rotating system for the purpose, the geocentric rectangular coordinate on the basis of J2000.0 mean equator is adopted. The direction of the X-axis is taken to the equinox of J2000.0 from the center of mass of the earth and the Z-axis is to the axis of the mean equator. The Y-axis is to be taken to make a right hand system by the X, Y and Z axes. This coordinate and other associated coordinates to express a position on the earth or in the space are as followings:

$X_{20}(X_{20}, Y_{20}, Z_{20})$ or simply $X(X, Y, Z)$: coordinate rectangular coordinate of the J2000.0 Mean equator,

$X_{TM}(X_{TM}, Y_{TM}, Z_{TM})$: Geocentric rectangular coordinate of the Mean equator of Date,

$X_{TT}(X_{TT}, Y_{TT}, Z_{TT})$: Geocentric rectangular Coordinate of the True equator of Date,

$U_P(U_P, V_P, W_P)$: Pseudo Earth-Fixed geocentric rectangular coordinate which does not account for the pole motion.

$U_E(U_E, V_E, W_E)$: Earth-Fixed geocentric coordinate whose reference plane is perpendicular to a line passing from the center of mass of the earth to the Conventional International Origin (CIO) and with U-axis passing through the Greenwich meridian.

$\Phi_G(\varphi, \lambda, h)$: Conventional geodetic coordinate referred an ellipsoid, namely, latitude, longitude and height from the ellipsoid.

$\Phi_L(R, Az, El)$: Topocentric spherical coordinate where R is distance from origin, Az is eastward azimuth from north and El is elevation from the horizontal plane.

The transformation matrixes from one coordinate to another for position and velocity on the coordinates described above are as followings:

i) Geocentric rectangular coordinate of the J2000.0 Mean equator ↔ Geocentric rectangular coordinate of the Mean equator of Date

$$\begin{cases} X_{TM} = PX_{20} \\ \dot{X}_{TM} = P\dot{X}_{20} \end{cases} \qquad \begin{cases} X_{20} = P^T X_{TM} \\ \dot{X}_{20} = P^T \dot{X}_{TM} \end{cases}$$

where it is assumed that $\dot{P}$ can be neglected practically and the expression of P is

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}$$

$$P_{11} = -\sin\zeta_\circ \sin z + \cos\zeta_\circ \cos z \cos\theta$$
$$P_{12} = -\cos\zeta_\circ \sin z - \sin\zeta_\circ \cos z \cos\theta$$
$$P_{13} = - \cos z \sin\theta$$
$$P_{21} = \sin\zeta_\circ \cos z + \cos\zeta_\circ \sin z \cos\theta$$
$$P_{22} = \cos\zeta_\circ \cos z - \sin\zeta_\circ \sin z \cos\theta$$
$$P_{23} = - \sin z \sin\theta$$
$$P_{31} = + \cos\zeta_\circ \sin\theta$$
$$P_{32} = - \sin\zeta_\circ \sin\theta$$
$$P_{33} = + \cos\theta$$

$$\zeta_\circ = 2306''.2181\,T + 0''.30188\,T^2 + 0''.017998\,T^3$$
$$z = 2306.2181\,T + 1.09468\,T^2 + 0.018203\,T^3$$
$$\theta = 2004.3109\,T - 0.42665\,T^2 - 0.041833\,T^3 \qquad \text{(Lieske } et\ al.\ \text{1977)}$$

and T is measured in Julian Centuries of 36525 days from JD 2451545.0 (2000 January 1.5).

ii) Geocentric rectangular coordinate of the Mean equator of Date ↔ Geocentric rectangular coordinate of the True equator of Date

$$\begin{cases} X_{TT} = NX_{TM} \\ \dot{X}_{TT} = N\dot{X}_{TM} \end{cases} \qquad \begin{cases} X_{TM} = N^T X_{TT} \\ \dot{X}_{TM} = N^T \dot{X}_{TT} \end{cases}$$

where it is assumed that $\dot{N}$ can be neglected practically and N is a matrix of the elments as

$$\begin{aligned}
N_{11} &= + \cos\Delta\psi \\
N_{12} &= -\cos\varepsilon_M \sin\Delta\psi \\
N_{13} &= -\sin\varepsilon_M \sin\Delta\psi \\
N_{21} &= + \cos\varepsilon_t \sin\Delta\psi \\
N_{22} &= \sin\varepsilon_M \sin\varepsilon_t + \cos\varepsilon_M \cos\varepsilon_t \cos\Delta\psi \\
N_{23} &= -\cos\varepsilon_M \sin\varepsilon_t + \sin\varepsilon_M \cos\varepsilon_t \cos\Delta\psi \\
N_{31} &= + \sin\varepsilon_t \sin\Delta\psi \\
N_{32} &= -\sin\varepsilon_M \cos\varepsilon_t + \cos\varepsilon_M \sin\varepsilon_t \cos\Delta\psi \\
N_{33} &= \cos\varepsilon_M \cos\varepsilon_t + \sin\varepsilon_M \sin\varepsilon_t \cos\Delta\psi
\end{aligned}$$

and

$$\begin{aligned}
\varepsilon_t &= \varepsilon_M + \Delta\varepsilon \\
\varepsilon_M &= 23^\circ 26' 21''.448 - 46''.8150\,T - 0''.00059\,T^2 + 0''.001813\,T^3 \\
\Delta\psi &= \sum_{i=1}^{106} (a_{0i} + a_{1i}T)\sin(b_{1i}l + b_{2i}l' + b_{3i}F + b_{4i}D + b_{5i}\Omega) \\
\Delta\varepsilon &= \sum_{i=1}^{106} (c_{0i} + c_{1i}T)\cos(f_{1i}l + f_{2i}l' + f_{3i}F + f_{4i}D + f_{5i}\Omega) \\
l &= 134^\circ 57' 46''.733 + (1325^r + 198^\circ 52' 02''.633)\,T + 31''.310\,T^2 + 0''.064\,T^3 \\
l' &= 357^\circ 31' 39''.804 + (90^r + 359^\circ 03' 01''.224)\,T - 0''.557\,T^2 - 0''.012\,T^3 \\
F &= 93^\circ 16' 18''.877 + (1342^r + 82^\circ 01' 03''.137)\,T - 13''.257\,T^2 + 0''.011\,T^3 \\
D &= 297^\circ 51' 01''.307 + (1236^r + 307^\circ 06' 41''.328)\,T - 6''.891\,T^2 + 0''.019\,T^3 \\
\Omega &= 125^\circ 02' 40''.280 - (5^r + 134^\circ 08' 10''.539)\,T + 7''.455\,T^2 + 0''.008\,T^3
\end{aligned}$$

T: Julian Centuries from JD 2451545.0

As for the a_{0i}, a_{1i}, b_{ji}, c_{0i}, c_{1i}, f_{ji}, consult Wahr (1979) or MERIT STANDARDS (Melbourne *et al.* 1983).

iii) Geodetic rectangular coordinate of the True equator of Date ↔ Pseudo Earth-Fixed geocentric rectangular coordinate

$$\begin{cases} U_p = SX_{TT} \\ \dot{U}_p = S\dot{X}_{TT} + \dot{S}X_{TT} \end{cases} \qquad \begin{cases} X_{TT} = S^T U_P \\ \dot{X}_{TT} = S^T \dot{U}_P + \dot{S}^T U_P \end{cases}$$

$$S = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \dot{S} = \begin{bmatrix} -\sin\theta & \cos\theta & 0 \\ -\cos\theta & -\sin\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \omega$$

$$\theta = 12^h + UT1 + \alpha_m + \Delta\psi\cos\varepsilon_t$$

$$\alpha_m = 24110^s\!.54841 + 8640184^s\!.812866\,T_U + 0^s\!.093104\,T_U^2 - 6^s\!.2\times10^{-6}\,T_U^3 \qquad \text{(Aoki } et\ al. \text{ 1981)}$$

$$T_U = d_U / 36525$$

d_U : the number of days of universal time elapsed since JD 2451545.0 (UT1)

$$d_U = d_{TAI} + (UT1 - TAI)^s / 86400$$

d_{TAI} : fraction of days of TAI elapsed since JD 2451545.0 (TAI)

ω : angular velocity of rotation of the earth

$$UT1 = UT1R + \Delta UT1$$

$$\Delta UT1 = \sum_{i=1}^{41} a_i \sin(g_{1i}l + g_{2i}l' + g_{3i}F + g_{4i}D + g_{5i}\Omega)$$

and $\Delta\psi$, ϵ_t, l, l', F, D and Ω are the same as the previous section. For a_i, g_{ji} Yoder (1981) or BIH Circular D (1982) should be referred.

iv) Pseudo Earth-Fixed geocentric rectangular coordinate ↔ Earth-Fixed geocentric rectangular coordinate

$$\begin{cases} U_E = BU_P \\ \dot{U}_E = B\dot{U}_P \end{cases} \qquad \begin{cases} U_P = B^T U_E \\ \dot{U}_P = B^T \dot{U}_E \end{cases}$$

where it is assumed that $\dot{B}$ can be neglected and the following expression of B is obtained from the strict formula in good approximation:

$$B = \begin{bmatrix} 1 & 0 & x_p \\ 0 & 1 & -y_p \\ -x_p & y_p & 1 \end{bmatrix}$$

If Q-matrix is defined by $Q = BSNP$, $U_E = QX_{20}$ and $X_{20} = Q^T U_E$ are the direct expression of the relation between X_{20} and U_E.

v) Earth-Fixed geocentric rectangular coordinate ↔ Conventional geodetic coordinate

$$U_E = (N_e + h_e)\cos\varphi\cos\lambda$$

$$V_E = (N_e + h_e)\cos\varphi\sin\lambda$$

$$W_E = (N_e(1-e^2) + h_e)\sin\varphi$$

$$N_e = A_e / (1 - e^2\sin^2\varphi)^{\frac{1}{2}}$$

where

A_e : semi-major axis of the reference ellipsoid
e : eccentricity of the reference ellipsoid
he : height from the reference ellipsoid.

(3) Acceleration due to the perturbation force

The acceleration due to the perturbation force, $\boldsymbol{R}$, in equation (1) is devided into a modeled term, $\boldsymbol{R}_m$, and an unmodeled term, $\boldsymbol{\eta}$, as

$$\boldsymbol{R}(r, v, t) = \boldsymbol{R}_m(r, v, t) + \boldsymbol{\eta}(r, v, t).$$

If the unmodeled term is small enough to be ignored or it can be assumed to be random and unbiased, the algorithm described above can be applied. The modeled terms in this processor are as followings:

$$\boldsymbol{R}_m = \alpha_{NS} + \alpha_{NB} + \alpha_{RP} + \alpha_{AD} + \alpha_{TD}$$

where

α_{NS}: non-spherical acceleration due to the earth gravity field

α_{NB}: difference of two-body accelerations to the satellite and to the center of mass of the earth due to the moon, the sun and planets

α_{RP}: acceleration due to the radiation pressure

α_{AD}: acceleration due to the atmospheric drag

α_{TD}: acceleration due to the solid earth tide and the ocean tide.

The expressions of these perturbations are as followings:

i) Non-spherical acceleration due to the earth gravity field

$$\alpha_{NS} = Q^T \alpha_{NS,E} \quad \text{(expressed in the non-rotating coordinate)}$$

$$\alpha_{NS,E} = -grad\ U \quad \text{(expressed in the earth-fixed coordinate)}$$

where U is the non-spherical gravity potential as

$$U = -\frac{\mu}{r}\left(\sum_{n=2}^{N}\left(\frac{A_e}{r}\right)^n J_n P_n(\sin\phi) + \sum_{n=2}^{N}\sum_{m=1}^{n}\left(\frac{A_e}{r}\right)^n P_n^m(\sin\phi)(C_{n,m}\cos m\lambda + S_{n,m}\sin m\lambda)\right).$$

These relations can be rewritten by using complex as followings (Cunningham 1970):

$$\alpha_{NS,E} = \text{Real}\left[\sum_{n=2}^{N}\sum_{m=0}^{n} A_e^n (C_{n,m} - iS_{n,m}) grad\ U_{n,m}\right]$$

where

$$\frac{\partial U_{n,m}}{\partial x} = -\frac{U_{n+1,m+1}}{2} + \frac{(n-m+2)!}{2(n-m)!} U_{n+1,m-1} \qquad (m>0)$$

$$= -\frac{U_{n+1,1}}{2} - \frac{U_{n+1,1}^*}{2} \qquad (m=0)$$

$$\frac{\partial U_{n,m}}{\partial y} = +\frac{iU_{n+1,m+1}}{2} + \frac{i(n-m+2)!}{2(n-m)!} U_{n+1,m-1} \qquad (m>0)$$

$$= \frac{iU_{n+1,1}}{2} - \frac{iU_{n+1,1}^*}{2} \qquad (m=0)$$

$$\frac{\partial U_{n,m}}{\partial z} = -\frac{(n-m+1)!}{(n-m)!} U_{n+1,m} \qquad (m \geqq 0)$$

$$U_{n,n} = (2n-1)\frac{(x+iy)}{r^2} U_{n-1,n-1}$$

$$U_{n,m} = (2n-1)\frac{z}{r^2} U_{n-1,m} \qquad (n=m-1)$$

$$U_{n,m} = \frac{(2n-1)}{(n-m)}\frac{z}{r^2} U_{n-1,m} - \frac{(n+m-1)}{(n-m)}\frac{U_{n-2,m}}{r^2} \qquad (n \neq m-1)$$

$$\left.\begin{aligned} x &= r\cos\phi\cos\lambda &&= U_E \\ y &= r\cos\phi\sin\lambda &&= V_E \\ z &= r\sin\phi &&= W_E \end{aligned}\right\} \text{earth-fixed coordinate of the satellite}$$

$$r = (U_E^2 + V_E^2 + W_E^2)^{\frac{1}{2}}$$

$$C_{n,m} = \left(\frac{k(2n+1)(n-m)!}{(n+m)!}\right)^{\frac{1}{2}} \bar{C}_{n,m}, \quad S_{n,m} = \left(\frac{k(2n+1)(n-m)!}{(n+m)!}\right)^{\frac{1}{2}} \bar{S}_{n,m} \quad k = \begin{cases} 1 : m=0 \\ 2 : m \neq 0 \end{cases}$$

The normalized harmonic coefficients $\bar{C}_{n,m}$ and $\bar{S}_{n,m}$, are given by Lerch *et al.* (1983) for GEM-L2 gravity model.

ii) Perturbed acceleration due to the moon, the sun and planets

$$\alpha_{NB} = -\mu \sum_{k=1}^{6} \frac{M_k}{M} \left(\frac{X_k}{X_k^3} - \frac{X_k - X}{|X_k - X|^3} \right)$$

where

X: position vector of the satellite

X_k: position vector of the moon (k=1), the sun (k=2), Venus (k=3), Mars (k=4), Jupiter (k=5) and Saturn (k=6)

$\frac{M_k}{M}$: mass ratio of these to the earth.

iii) Acceleration due to the radiation pressure

$$\alpha_{RP} = \nu P_S A_A^2 \frac{\gamma A_S}{m} \frac{(X - X_2)}{|X - X_2|^3}$$

where

X, X_2: position vector of the satellite and the sun

$\nu = \begin{cases} 1\text{: at out of the shadow by the earth} \\ 0\text{: in the shadow by the earth} \end{cases}$

$P_S =$ 4.5605 x 10^{-6} Newtons/m^2: solar radiation pressure at $|X - X_2| = A_A$ (Melbourne *et al.* 1983)

$A_A =$ 1.49597870 x 10^{11}m: astronomical unit (IAU 1976)

γ: reflectivity coefficient

A_S: cross section area of the satellite to $(X - X_2)$

m: mass of the satellite

iv) Acceleration due to the atmospheric drag

$$\alpha_{AD} = -\beta \rho v_r \mathbf{v}_r$$

where

$\mathbf{v}_r = \mathbf{v} - \boldsymbol{\omega} \times X$ and $v_r = |\mathbf{v}_r|$

$\beta = \frac{C_d A_s}{2m}$: ballistic coefficient given by Melbourne *et al.* (1983) for each satellite

$\rho = \rho_0 e^{-k(r - A_{me} - h_0)}$: atmospheric density

X, V: position and velocity vector of the satellite in the non-rotating coordinate

ω: rotation vector of the earth in the non-rotating coordinate

r: geocentric distance to the satellite

A_{me}: mean radius of the earth (6371 km)

ρ_0: atmospheric density at height h_0. Refer to e.g. Allen (1973).

k: scale constant of the atmospheric density

v) Acceleration due to the solid earth tide and the ocean tide

$$\alpha_{TD} = -grad\ \Delta U_{TD}$$

The acceleration by the effects of solid earth tide and ocean tide associated with ΔU_{TD} is calculated through the variation of geopotential coefficients, $\bar{C}_{nm}$ and $\bar{S}_{n,m}$, as followings (Melbourne *et al.* 1983):

For the first step, the corrections to add to the coefficients by the solid earth tide are:

$$\Delta\bar{C}_{2,0} = \frac{1}{\sqrt{5}} k_2 \frac{A_e^3}{\mu} \sum_{k=1}^{2} \frac{GM_k}{r_k^3} P_2^0(\sin\phi_k)$$

$$\Delta\bar{C}_{2,1} - i\Delta\bar{S}_{2,1} = \frac{1}{3}\sqrt{\frac{3}{5}} k_2 \frac{A_e^3}{\mu} \sum_{k=1}^{2} \frac{GM_k}{r_k^3} P_2^1(\sin\phi_k) e^{-i\lambda_k}$$

$$\Delta\bar{C}_{2,2} - i\Delta\bar{S}_{2,2} = \frac{1}{12}\sqrt{\frac{12}{5}} k_2 \frac{A_e^3}{\mu} \sum_{k=1}^{2} \frac{GM_k}{r_k^3} P_2^2(\sin\phi_k) e^{-i2\lambda_k}$$

where

k_2: nominal second degree Love number

(r_k, ϕ_k, λ_k): earth-fixed geocentric spherical coordinate of the moon (k=1) and the sun (k=2).

Other notations are the same as described above.

For the second step,

$$\Delta\bar{C}_{n,m} - i\Delta S_{n,m} = F_m \sum_{S(n,m)} \delta k_s H_s \binom{1}{-i}_{n-m\ odd}^{n-m\ even} e^{i\theta_s}$$

where

$$F_m = \frac{(-1)^m}{A_e\sqrt{4\pi(2-\delta_{0m})}}, \qquad \delta_{0m} = \begin{cases} 1 & m=0 \\ 0 & m\neq 0 \end{cases}$$

δk_s: difference between Wahr model for k at frequency s and the nominal value k_2 in the sense $k_s - k_2$

H_s: amplitude of term at frequency s from a harmonic expansion of the tide generating potential,

$\theta_s = \bar{n}\cdot\bar{\beta} = \sum_{i=1}^{6} n_i\beta_i$

$\bar{n}$ six multipliers of the Doodson variables

$\bar{\beta}$ the Doodson variables

$\delta S_{20} = 0$.

The Doodson variables are related to the fundamental arguments of the nutation described above by

$$s = F + \Omega = \beta_2$$
$$h = s - D = \beta_3$$
$$p = s - l = \beta_4$$
$$N' = -\Omega = \beta_5$$
$$P_1 = s - D - l' = \beta_6$$
$$\tau = \theta_g + \pi - s = \beta_1$$

θ_g : mean siderial time of the conventional zero meridian

For $F_m k_s H_s$ and $\bar{n}$, see Melbourne *et al.* (1983).

As for the effect by ocean tide, the correction to the coefficients of geopotential are calculated as followings:

$$\Delta\bar{C}_{n,m} - i\Delta\bar{S}_{n,m} = B_{nm} \sum_{s(n,m)} \sum_{+}^{-} (C^{\pm}_{snm} \mp iS^{\pm}_{snm}) e^{\pm i\theta_s}$$

where

$$B_{nm} = \frac{4\pi G\rho_W}{g} \left(\frac{(n+m)!}{(n-m)!(2n+1)(2-\delta_{0m})} \right)^{\frac{1}{2}} \frac{1+k'_n}{2n+1}$$

$g = GM/A_e^2$

ρ_w density of seawater

k'_n load deformation coefficients

G the universal gravitational constant

$C^{\pm}_{snm}$, $S^{\pm}_{snm}$: ocean tide coefficients in m for the tide constituent s .

For θ_s, the notation is the same as before. As for the values of $C^{\pm}_{snm}$, $S^{\pm}_{snm}$ and k'_n, see the table in Melbourne *et al.* (1983).

(4) Expression of A-matrix

The unknowns of this estimation process are denoted by X as already described. For the unknowns, satellite position, X, satellite velocity, V, and any astrogeodetic parameters which are not weakly associated with satellite orbit can be taken, e.g., geocentric constant of gravitation $\mu(= GM)$,dynamical form factor for the earth $J_2(= -C_{2,0}>0)$, ballistic coefficient $\beta(= C_d A_s/2m)$, reflectivity coefficient γ, pole position (x_p, y_p) and all observation site coordinates except a longitude of an observation site in the case of the range and range rate observation of satellite. In this estimation procedure the following variables are taken as the unknowns:

$$X = [X,\ V,\ \mu,\ J_2,\ \beta,\ U_1,\ U_2,\ \cdots\cdots,\ U_N]^T$$
$$\dot{X} = F = [V,\ \alpha,\ 0,\ 0,\ 0,\ 0,\ 0,\ \cdots\cdots,\ 0]^T .$$

The expression of the A-matrix in equation (6) is:

$$A=\left[\frac{\partial F}{\partial X}\right]^*=\begin{bmatrix} 0_3, & I_3, & 0, & 0, & 0, & 0_3, & 0_3, & \cdots, & 0_3 \\ \frac{\partial \alpha}{\partial X}, & \frac{\partial \alpha}{\partial V}, & \frac{\partial \alpha}{\partial \mu}, & \frac{\partial \alpha}{\partial J_2}, & \frac{\partial \alpha}{\partial \beta}, & 0_3, & 0_3, & \cdots, & 0_3 \\ 0_3, & 0_3, & 0, & 0, & 0, & 0_3, & 0_3, & \cdots, & 0_3 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \\ 0_3, & 0_3, & 0 & 0, & 0, & 0_3, & 0_3, & \cdots, & 0_3 \end{bmatrix}$$

where

$$\alpha=\alpha_{2B}+(\alpha_{NS}+\alpha_{TD})+\alpha_{NB}+\alpha_{RP}+\alpha_{AD}$$

$$\alpha_{2B}=-\mu\frac{X}{r^3}$$

$$0_3=\begin{bmatrix} 0, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \end{bmatrix} \quad \text{and} \quad I_3=\begin{bmatrix} 1, & 0, & 0 \\ 0, & 1, & 0 \\ 0, & 0, & 1 \end{bmatrix}.$$

i) Expression of $\partial \alpha_{2B}/\partial X$

$$\frac{\partial \alpha_{2B}}{\partial X}=\begin{bmatrix} \frac{\partial \alpha_{2Bx}}{\partial X} & \frac{\partial \alpha_{2Bx}}{\partial Y} & \frac{\partial \alpha_{2Bx}}{\partial Z} \\ \frac{\partial \alpha_{2By}}{\partial X} & \frac{\partial \alpha_{2By}}{\partial Y} & \frac{\partial \alpha_{2By}}{\partial Y} \\ \frac{\partial \alpha_{2Bz}}{\partial X} & \frac{\partial \alpha_{2Bz}}{\partial Y} & \frac{\partial \alpha_{2Bz}}{\partial Z} \end{bmatrix}=\frac{\mu}{r^3}\begin{bmatrix} \frac{3X^2}{r^2}-1, & \frac{3XY}{r^2} & \frac{3XZ}{r^2} \\ \frac{3XY}{r^2} & \frac{3Y^2}{r^2}-1, & \frac{3YZ}{r^2} \\ \frac{3XZ}{r^2} & \frac{3YZ}{r^2} & \frac{3Z^2}{r^2}-1 \end{bmatrix}$$

or in the expression of elements,

$$\frac{\partial \alpha_{2B}}{\partial X}\Big|_{ij}=\frac{\partial \alpha_{2B_i}}{\partial X_j}=\frac{\mu}{r^3}\left\{\frac{3X_iX_j}{r^2}-\delta_{ij}\right\}.$$

ii) Expression of $\partial(\alpha_{NS}+\alpha_{TD})/\partial X$

The effects of the solid earth tide and the ocean tide are included in the corrected harmonic coefficients of geopotential as described before. It is assumed that the coefficients in this paragraph include the tidal effects.

By taking gradient of $\alpha_{NS}=Q^T\alpha_{NS,E}$, the relation of transformation from the earth-fixed geocentric rectangular coordinate to the non-rotating coordinate as followings are obtained:

$$\frac{\partial \alpha_{NS}}{\partial X}=Q^T\frac{\partial \alpha_{NS,E}}{\partial U}Q.$$

Using the similar notation to the paragraph of the acceleration due to the earth gravity field described before, the following relations are given by Cunningham (1970):

$$\frac{\partial \alpha_{NS,E}}{\partial U}=-\frac{\partial}{\partial U}\frac{\partial U}{\partial U}$$

or using (x, y, z) instead of (U, V, W) for earth-fixed geocentric rectangular coordinate and notation of matrix element,

$$\frac{\partial \alpha_{NS,E}}{\partial \boldsymbol{x}}\Big|_{ij} = -\frac{\partial^2 U}{\partial x_i \partial y_i} = \text{Real}\sum_{n=2}^{N}\sum_{m=0}^{n} A_e^n (C_{n,m} - iS_{n,m})\frac{\partial^2 U_{n.m}}{\partial x_i \partial y_j}$$

where

$$\frac{\partial^2 U_{n,m}}{\partial x^2} = \frac{U_{n+2,m+2}}{4} - \frac{(n-m+2)!}{2(n-m)!}U_{n+2,m} + \frac{(n-m+4)!}{4(n-m)!}U_{n+2,m-2} \qquad m>1$$

$$= \frac{U_{n+2,3}}{4} - \frac{(n+1)!}{2(n-1)!}U_{n+2,1} - \frac{(n+1)!}{4(n-1)!}U^*_{n+2,1} \qquad m=1$$

$$= \frac{U_{n+2,2}}{4} - \frac{(n+2)!}{2n!}U_{n+2,0} + \frac{U^*_{n+2,2}}{4} \qquad m=0$$

$$\frac{\partial^2 U_{n,m}}{\partial x \partial y} = -\frac{iU_{n+2,m+2}}{4} + \frac{i(n-m+4)!}{4(n-m)!}U_{n+2,m-2} \qquad m>1$$

$$= -\frac{iU_{n+2,3}}{4} - \frac{i(n+1)!}{4(n-1)!}U^*_{n+2,1} \qquad m=1$$

$$= -\frac{iU_{n+2,2}}{4} + \frac{iU^*_{n+2,2}}{4} \qquad m=0$$

$$\frac{\partial^2 U_{n,m}}{\partial y^2} = -\frac{U_{n+2,m+2}}{4} - \frac{(n-m+2)!}{2(n-m)!}U_{n+2,m} - \frac{(n-m+4)!}{4(n-m)!}U_{n+2,m-2} \qquad m>1$$

$$= -\frac{U_{n+2,3}}{4} - \frac{(n+1)!}{2(n-1)!}U_{n+2,1} + \frac{(n+1)!}{4(n-1)!}U^*_{n+2,1} \qquad m=1$$

$$= -\frac{U_{n+2,2}}{4} - \frac{(n+2)!}{2n!}U_{n+2,0} - \frac{U^*_{n+2,2}}{4} \qquad m=0$$

$$\frac{\partial^2 U_{n,m}}{\partial x \partial z} = \frac{(n-m+1)}{2}U_{n+2,m+1}, - \frac{(n-m+3)!}{2(n-m)!}U_{n+2,m-1} \qquad m>0$$

$$= \frac{(n+1)}{2}U_{n+2,1} + \frac{(n+1)}{2}U^*_{n+2,1} \qquad m=0$$

$$\frac{\partial^2 U_{n,m}}{\partial y \partial z} = -\frac{i(n-m+1)}{2}U_{n+2,m+1} - \frac{i(n-m+3)!}{2(n-m)!}U_{n+2,m-1} \qquad m>0$$

$$= -\frac{i(n+1)}{2}U_{n+2,1} + \frac{i(n+1)}{2}U^*_{n+2,1} \qquad m=0$$

$$\frac{\partial^2 U_{n,m}}{\partial z^2} = +\frac{(n-m+2)!}{(n-m)!}U_{n+2,m}. \qquad m\geq 0$$

iii) Expression of $\partial\boldsymbol{\alpha}_{NB}/\partial\boldsymbol{X}$, $\partial\boldsymbol{\alpha}_{RP}/\partial\boldsymbol{X}$ and $\partial\boldsymbol{\alpha}_{AD}/\partial\boldsymbol{X}$

$$\frac{\partial \alpha_{NB}}{\partial \boldsymbol{X}}\Big|_{ij} = \mu\sum_{k=1}^{6}\frac{M_k}{M}\frac{1}{R_k^3}\frac{3(X_i - X_{ki})(X_j - X_{kj})}{R_k^2} - \delta_{ij}$$

where

$$R_k = \{(X_1 - X_{k1})^2 + (X_2 - X_{k2})^2 + (X_3 - X_{k3})^2\}^{\frac{1}{2}}.$$

$$\frac{\partial \alpha_{RP}}{\partial \boldsymbol{X}}\Big|_{ij} = \nu P_S A_A^2 \frac{\gamma A_S}{m}\frac{1}{R_2^3}\left\{\delta_{ij} - \frac{3(X_i - X_{2i})(X_j - X_{2j})}{R_2^2}\right\}.$$

$$\frac{\partial \alpha_{AD}}{\partial \boldsymbol{X}}\Big|_{ij} = -\beta\rho\left\{-\frac{kv_r}{r}(v_{r_i}X_j + \frac{1}{v_r}v_{r_i}(\boldsymbol{\omega}\times\boldsymbol{v}_r)_j - v_r\Omega_{ij}\right\},$$

where

$$r = (X_1^2 + X_2^2 + X_3^2)^{\frac{1}{2}}$$

$$\Omega=\begin{bmatrix} 0 & \omega_z & -\omega_y \\ -\omega_z & 0 & \omega_x \\ \omega_y & -\omega_x & 0 \end{bmatrix}$$

$$v_{r_i}=(V-\omega\times X)_i \quad \text{and} \quad v_r=(v_{r1}^2+v_{r2}^2+v_{r3}^2)^{\frac{1}{2}}$$

iv) Expression of $\partial\alpha/\partial V$

$$\frac{\partial\alpha}{\partial V}\Big|_{ij}=\frac{\partial\alpha_{AD}}{\partial V}\Big|_{ij}=-\beta\rho\frac{1}{v_r}(v_{r_i}v_{r_j}+\delta_{ij}v_r^2).$$

v) Expression of $\partial\alpha/\partial\mu$, $\partial\alpha/\partial J_2$ and $\partial\alpha/\partial\beta$

$$\frac{\partial\alpha}{\partial\mu}=-\frac{X}{r^3}+\frac{1}{\mu}\alpha_{NS}.$$

$$\frac{\partial\alpha}{\partial J_2}=\frac{1}{J_2}\alpha_{J_2}.$$

where α_{J_2} can be obtained from the J_2 term of α_{NS},

$$\frac{\partial\alpha}{\partial\beta}=\frac{1}{\beta}\alpha_{AD}.$$

4. Observations of dynamical system

(1) Observation-state relationships

The range to a satellite from a j-th observation site, R, is

$$R_j=\{(U-U_j)^2+(V-V_j)^2+(W-W_j)^2\}^{\frac{1}{2}}=\{(X-X_j)^2+(Y-Y_j)^2+(Z-Z_j)^2\}^{\frac{1}{2}}$$

where $(U,\ V,\ W)$ is the coordinate of satellite position and $(U_j,\ V_j,\ W_j)$ is the j-th site position in the earth-fixed geocentric rectangular coordinate. $(X,\ Y,\ Z)$ and $(X_j,\ Y_j,\ Z_j)$ are also the satellite position and site position expressed in the non-rotating coordinate, respectively. To say in the strict sense in this estimation procedure, $(U,\ V,\ W)$ or $(X,\ Y,\ Z)$ should be the position of the center of mass of the satellite and $(U_j,\ V_j,\ W_j)$ or $(X_j,\ Y_j,\ Z_j)$ should be the position of the reference point of the satellite laser ranging system at the j-the observation site.

The station position expressed in the earth-fixed coordinate has been treated as constant in this procedure. Though the real station position on the earth is periodically moved slightly by the solid earth tide and ocean tide loading, the constant coordinate of the j-th site, $(U_j,\ V_j,\ W_j)$, can be regarded as mean position. The site displacement by the tides is added to the mean position. The raw range to the satellite observed by a laser ranging system contains the effects of atmospheric refraction, difference between positions of reference point of reflectors and satellite center of mass and individual range offset for each ranging system. If Y denotes the raw range, the G-function relating the state and observation is expressed from equation (4) as followings:

$$Y=R+\Delta R_{DP}+\Delta R_{RF}+\Delta R_{CM}+\Delta R_{RO}+\varepsilon$$

or

$$G=R+\Delta R_{DP}+\Delta R_{RF}+\Delta R_{CM}+\Delta R_{RO}$$

where

R : distance from a mean reference point of a laser ranging system at the site to the center of mass of the satellite

ΔR_{DP} : component of tidal displacements along the direction from site to satellite

ΔR_{RF} : change of range by atmospheric refraction

ΔR_{CM} : satellite center of mass correction

ΔR_{RO} : range offset for laser ranging system obtained by system calibration.

(2) Site displacements by the solid earth tide and the ocean tide loading

The site displacement caused by the solid earth tide is estimated in two-step procedure. The vector displacement of the j-th site due to tidal deformation for step 1 can be computed by the formula

$$\Delta U_j=\sum_{k=1}^{2}\frac{GM_k r_j^4}{\mu r_k^3}\left[\{3l_2(\boldsymbol{u}_k\cdot\boldsymbol{u}_j)\}\boldsymbol{u}_k+\left\{3\left(\frac{h_2}{2}-l_2\right)(\boldsymbol{u}_k\cdot\boldsymbol{u}_j)^2-\frac{h_2}{2}\right\}\boldsymbol{u}_j\right]$$

where

r_k, u_k : magnitude of the vector from the geocenter to the moon(k=1) or to the sun(k=2) and unit vector of the vector

r_j, u_j : magnitude of the vector from the geocenter to the j-th site and unit vector of the vector

h_2 : nominal second degree Love number

l_2 : nominal Shida number.

For the step 2, only the displacement of one term K_1 (165.555 in Doodson number) frequency needs to be corrected as a periodic change in site height given by

$$\delta h_j^{(1)}=\delta h_{K_1}\ H_{K_1}\left(-\sqrt{\frac{5}{24\pi}}\right)3\ \sin\phi_j\ \cos\phi_j\ \sin(\theta_{K_1}+\lambda_j)$$

where

$\delta h_{K_1}=h_{K_1}$ (Wahr) $- h_2$ (Nominal)

H_{K_1} = amplitude of K_1 term in the harmonic expansion of the tide generating potential

ϕ_j = geocentric latitude of j-th site

λ_j = east longitude of j-th site

θ_{K_1} = K_1 tide argument $= \tau + s = \theta_g + \pi$.

For the values of h_2, l_2, δh_{K_1} H_{K_1} consult e.g. Melbourne *et al.* (1983).

There is also a zero frequency site displacement. The correction could be removed analogously to the discussion above. If nominal Love and Shida numbers of 0.6090 and 0.0852, respectively, are used, the permanent deformation introduced in the height direction is given by

$$\delta h_j^{(2)}=\sqrt{\frac{5}{4\pi}}(0.6090)(-0.31455)\left(\frac{3}{2}\sin^2\phi_j-\frac{1}{2}\right)\qquad\text{(meter)}$$

and in the north direction in the horizontal plane

$$\delta n_j^{(1)}=\sqrt{\frac{5}{4\pi}}(0.0852)(-0.31455)\ 3\cos\phi_j\ \sin\phi_j \qquad \text{(meter)}.$$

The site displacements caused by the ocean tide loading have been computed for the *M2, S2, K2, N2, O1, K1, P1, Q1* and *Mf* ocean tides by Goad (1980) using models generated by Schwiderski (1978). The tables of tidal loading height displacement amplitude and phase values of 25 laser site locations for the nine constituents are listed in Melbourne *et al.* (1983). The height displacement at *j*-th site can be obtained by summing nine constituents as follows:

$$\delta h_j^{(0)}=\sum_{i=1}^{9} \text{amp}_j(i)\cos(\text{arg}_j(i,t)-\text{phase}(i))$$

where $arg_j(i, t)$ can be also generated by the subroutine in the same paper.

As for the Simosato site Sasao and Kikuchi (1982) calculated the three components of the ocean tide loading site displacement of height, from south to north and from west to east for the nine constituents by using 1° × 1° lattice of Schwiderski model, 232 points on the coast line and tidal data at six tidal stations around the Simosato site. The results are shown in Table 2. The sense of amplitude sign is that up in height, to north and to east are all +. The phase and argument are the same meaning as the equation described above.

Table 2 Ocean tide loading site displacement for Simosato site by Sasao and Kikuchi (1982)

constituent	vertical		from south to north		from west to east	
	amp (cm)	phase (deg)	amp (cm)	phase (deg)	amp (cm)	phase (deg)
M 2	1.817	86.6	0.366	108.7	0.276	−164.2
S 2	0.803	106.5	0.147	129.3	0.143	−131.1
K 2	0.237	109.5	0.043	130.3	0.037	−126.9
O 1	1.073	−146.4	0.160	−113.7	0.155	−8.4
K 1	1.388	−126.1	0.215	−93.9	0.191	14.6
P 1	0.445	−127.0	0.067	−93.7	0.062	15.4
N 2	0.360	91.6	0.070	101.0	0.039	−161.8
Q 1	0.228	−155.2	0.032	−129.0	0.033	−13.2
M f	0.021	−26.9	0.002	−109.4	0.005	115.4

Latitude of Simosato site = 33°.578, longitude = 135°.937, height = 62m.

Finally the range correction ΔR_{DP}, for *j*-th site are

where $$\Delta R_{DP}=[\Delta U_j+(\delta h_j^{(1)}+\delta h_j^{(2)}+\delta h_j^{(0)}h_j+(\delta n_j^{(1)}+\delta n_j^{(0)}n_j+\delta e_j^{(0)}e_j]\cdot\frac{(u_j-u)}{|u_j-u|}$$

u_j : unit vector from geocenter to *j*-th site
u : unit vector from geocenter to satellite
h_j : unit vector from *j*-th site to the zenith
n_j : unit vector from *j*-th site to the north in the horizontal plane
e_j : unit vector from *j*-th site to the east in the horizontal plane.

(3) Correction of the atmospheric refraction

The following formula of the atmospheric correction to the laser range data given by Marini and Murray (1973) is used in this processor:

$$\Delta R_{RF}=\frac{g(\lambda)}{f(\varphi,H)}\cdot\frac{A+B}{\sin E+\dfrac{B/(A+B)}{\sin E+0.01}}$$

where

$$g(\lambda)=0.9650+\frac{0.0164}{\lambda^2}+\frac{0.000228}{\lambda^4}$$
$$f(\varphi,H)=1-0.0026\cos 2\varphi-0.00031H$$
$$A=0.002357P+0.000141e$$
$$B=(1.084\times 10^{-8})PTK+(4.734\times 10^{-8})\frac{P^2}{T}\frac{2}{(3-1/K)}$$
$$K=1.163-0.00968\cos 2\varphi-0.00104T+0.00001435P$$
$$e=6.11\cdot\frac{Rh}{100}\cdot 10^{7.5(T-273.15)/(237.3+(T-273.15))}$$

Here

ΔR_{RF} : Range correction (meters)
E: True elevation of satellite
P: Atmospheric pressure at the laser site (millibars)
T: Atmospheric temperature at the laser site (degrees Kelvin)
Rh : Relative humidity at the laser site (%)
λ: Wavelength of the laser (microns)
φ: Latitude of the laser site
H: Altitude of the laser site (kilometers).

(4) Other corrections and expression of ∂G/∂X

The center of mass correction should be applied for each satellite. The values are:

ΔR_{CM} = –0.24m for Lageos and –0.075m for Starlette.

As for the range offset for each laser ranging system, it is usually obtained by ground target ranging or internal calibration techniques. The values for the satellite laser ranging system at Simosato site, for instance, are distributed from 10 to 25 centimeters which depend on the energy of output and input and environmental conditions.

The expression for *G*-function was given in the previous section. In the relation between observed range and nominal range, Y or Y_j and R or R_j, all the correction described above should be considered. However the dependence of these correction to the selected unknown, X, is nothing or so small. Therefore these corrections are ignored for $\partial G/\partial X$ in this estimation.

The expression for $\partial G/\partial X$ is as followings:

$$\frac{\partial G}{\partial X}=\frac{X^T-X_j^T}{R_j}$$

$$\frac{\partial G}{\partial V}=0^T$$

$$\frac{\partial G}{\partial \mu}=\frac{\partial G}{\partial J_2}=\frac{\partial G}{\partial \beta}=0$$

$$\frac{\partial G}{\partial U_i}=-\frac{U^T-U_j^T}{R_j}\delta_{ij}$$

where the definitions of these variables are the same as described in this chapter.

5. Processing of the laser range data and preliminary results

(1) Range data processing and constants used

The station coordinate of Simosato site is estimated using the estimation procedure described above by the processor developed. For the estimation, 756 ranges of satellite laser data in 24 passes of Lageos transit are used as listed in Table 3. These data are obtained at Simosato site and other six sites which are distributed in Australia, north America, Central Pacific Ocean and Europe. As the amount of data used are not so much in this time, the number of unknowns is limited to nine variables, namely, satellite position and velocity at an initial time and the coordinate of Simosato site. So, the positions of the six sites except Simosato are given a priori. For the coordinate system of the six sites the LPM 81.12 (Schutz 1983a) is adopted. This coordinate system has been used at the Center for Space Research of the University of Texas to estimate the earth rotation parameters, and the every five days values of x and y component of pole position and the Duration of Day are announced in the CSR reports and in the BIH Circular D. The adopted coordinates of the six sites on the basis of the LPM 81.12 system are given in Table 4.

For the coefficients of the gravity field model, GEM-L2 (Lerch *et al.* 1983) is used. In the gravity model the terms of $\bar{C}_{2,1}$ and $\bar{S}_{2,1}$, which are caused by the discrepancy of the polar axis of the terrestrial system and the principal axis of moment of inertia by using CIO, have values as

$$\bar{C}_{2,1}=1.057\times10^{-9} \quad \text{and} \quad \bar{S}_{2,1}=-3.068\times10^{-9}.$$

To save computation time the coefficients less than 11 degrees and 11 orders which include major effective terms are used in this estimation.

For the UT1R and x, y components of the earth rotation BIH evaluation in the Circular D (BIH 1982) are used.

The mass ratios of the moon, the sun, Venus, Mars, Jupiter and Saturn to the earth of IAU 1976 system are used. For the positions of these astronomical bodies a magnetic tape file of the Japanese Ephemeris (JHD 1980) is used. In the transformation from the 1950.0 frame used in the ephemeris to J2000.0, the correction from FK5 equinox (J2000.0) to FK4 equinox as following by Fricke (1980) is applied:

$$E(T)=0^{s}.035+0^{s}.085\,T_{50}$$

Table 3 Lageos date set used for the coordinate determination of Simosato site

No. pass	U T C d h m s h m s	Site ID	No. of ranges
	April 1982		
1	8 3 3 33 — 3 12 13	1	24
2	8 7 51 15 — 8 15 39	4	39
3	8 11 24 40 — 11 54 41	4	40
4	9 9 55 6 — 10 36 36	4	40
5	9 17 56 33 — 18 10 33	1	25
6	10 16 29 34 — 16 37 40	1	17
7	11 18 45 15 — 18 53 22	1	14
8	13 8 1 27 — 8 38 18	4	40
9	13 12 33 56 — 12 36 32	5	16
10	13 16 29 26 — 16 35 1	2	40
11	14 2 37 44 — 3 0 44	6	28
12	14 14 59 43 — 15 36 35	2	40
13	14 18 28 54 — 19 10 0	2	40
14	15 1 15 24 — 1 39 28	6	28
15	15 1 32 28 — 1 36 28	7	37
16	15 4 54 52 — 5 9 20	6	11
17	15 5 2 24 — 5 12 36	7	36
18	15 5 12 43 — 5 46 44	3	40
19	15 8 25 5 — 8 26 36	7	10
20	15 8 56 11 — 8 59 40	3	40
21	15 9 9 57 — 9 33 22	4	40
22	15 13 3 34 — 13 26 10	5	40
23	15 16 45 54 — 16 58 11	1	31
24	15 17 1 2 — 17 48 56	2	40

where T_{50} is measured in Julian centuries from 1950.0.

The following constants are used in this estimation:

Light velocity; $c = 2.99792458 \times 10^8$ m/s
Geocentric constant of gravitation; $GM = 398600.44$ km^3/s^2
Lageos mass; $m = 407.8$ kg
Lageos cross section areas; $A_s = 0.283$ m^2
Lageos reflectivity coefficient; $\gamma = 1.17$
Lageos atmospheric drag coefficient; $C_d = 3.8$
Lageos center of mass correction; $R_{CM} = -0.24$ m
Lageos empirical acceleration; $\alpha_{ep} = -2.9 \times 10^{-12}$ m/s^2
Atmospheric density at $h = 5000$ km; $\rho_0 = 3.98 \times 10^{-5}$ kg/km^3
Scale constant of the atmospheric density; $k = 1.61 \times 10^{-3}$/km.

Table 4 Station coordinates by LPM81.12

ID	name/state	Lat. Lon. Ht
1	Simosato/Japan	(unknown)
2	Yaragadee/Australia	29° 2′ 47″.4692 S 115 20 48.0579 E 244.960 m
3	Greenbelt/Maryland	39 1 14.1748 N 283 10 20.1161 E 21.985 m
4	Platteville/Colorado	40 10 58.0085 N 255 16 26.2849 E 1504.807 m
5	Mt. Haleakala/Hawaii	20 42 25.9795 N 203 44 38.5366 E 3068.264 m
6	Kootwijk/Netherlands	52 10 42.2302 N 5 48 35.0936 E 93.025 m
7	Wettzell/West Germany	49 8 41.7703 N 12 52 40.9405 E 660.988 m

Reference ellipsoid : A=6378137.0m
f=1/298.255

(2) Coordinate of Simosato site and relation between the LPM 81.12 system and the Tokyo Datum

The values of unknowns are obtained by integrating X^* and Φ and by evaluating H and y. The results for nine unknowns are:

$$X_0 = (8436.59040^{km}, 8281.56187^{km}, 3440.96763^{km})^T$$
$$V_0 = (0.329606460^{km/s}, -2.482641673^{km/s}, 5.098940310^{km/s})^T$$
$$\text{at } t = 1982^{Y}\ 04^{M}\ 08^{D}\ 03^{h}\ 00^{m}\ 20^{s}.000000 \text{ (TAI) and}$$
$$U_1 = (-3822.38450^{km}, 3699.36641^{km}, 3507.57257^{km})^T.$$

The residuals of range data based on the initial values and site coordinate are shown in Figure 1. The root mean square (RMS) of the residuals is 55 cm. The position of the intersecting point of the elevation and azimuth axes of the satellite laser ranging system, U_1, based on the LPM 81.12 system can be rewritten in the conventional geodetic coordinate as:

33° 34′ 39″. 697N	(latitude)
135 56 13 . 156E	(longitude)
100 . 66 m	(height from the reference ellipsoid)

where semi-major axis and flattening of the reference ellipsoid are A_e = 6378137.0 m, and $1/f$ = 298.257.

The surveyed coordinate of the same point in the Tokyo Datum (Takemura 1982) is:

33° 34′ 27″.496N	(latitude)
135 56 23.537E	(longitude)
62.44 m	(height above mean sea level).

The direct comparison of both heights derives geoidal height based on the reference ellipsoid as 38.2 m at Simosato site. If the geoidal height of the Tokyo Datum at Simosato site is estimated as 0.0 m by a geoid map of Ganeko (1976), the comparison of two coordinate values of the LPM 81.12 and the Tokyo Datum for the same point at Simosato site derives the datum shift correction for geocentric rectangular coordinate from the Tokyo Datum to the LPM 81.12 system as following:

$$\Delta U = -142.5 \text{ m}$$
$$\Delta V = +510.4 \text{ m}$$
$$\Delta W = +681.2 \text{ m}$$

where the U-, V- and W-axes in both the LPM 81.12 system and the Tokyo Datum are assumed to be parallel for each axes. The datum shift correction derives the expression of the position of the origin of the Tokyo Datum in the LPM 81.12 system as followings:

35° 39′ 29″.223N	(latitude)
139 44 28.676E	(longitude)
62.95 m	(height from the reference ellipsoid).

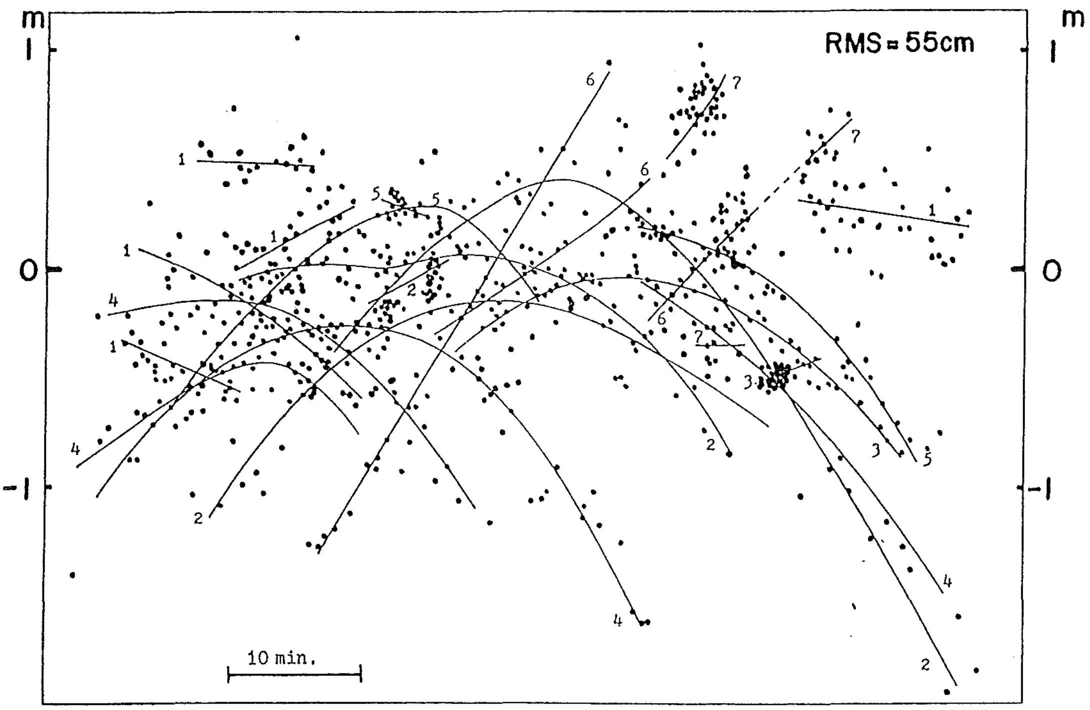

Figure 1 Residuals to the orbit of Lageos determined by the processor. Solid lines show centers of dispersed ranges of each passes with site ID.

As the height above mean sea level of the origin is known as 26.80 m, the geoidal height at the origin is estimated as 36.1 m on the basis of the reference ellipsoid of A_e = 6378137.0 m and $1/f$= 298.257. The discrepancy in both coordinate systems for the position of the origin amounts to 468 m to the direction of 321° in azimuth on the basis of the Tokyo Datum. The similar discrepancies can be obtained using the datum shift correction to the place at Simosato, Sapporo, Kagoshima, for instance, as 462 m to 325° of azimuth, 411 m to 311° and 448 m to 330°, respectively.

Within the procedure of orbital determination by satellite laser ranging data or Doppler data, the longitude of a satellite-derived coordinate system and the position of the ascending node of the satellite orbit can not be separated. So, it is necessary to define the longitude of one satellite ranging site a priori. According to Schutz (1983c) 283° 10′ 19.″7510 is the definition of the longitude of the LPM 81.12 system for the longitude of the reference point of the satellite laser ranging system named STALAS at the Goddard Space Flight Center. So, the longitude of Simosato site shown above is based on the definition. It is expected to be combined with such a satellite derived longitude system and the precise astronomical longitude like the lunar laser ranging (LLR) which has been operated at the McDonald Observatory, the University of Texas. As for the relation between the LLR results and the LPM 81.12 system in longitude, Schutz (1983c) informed to the author that an eastward rotation of the LPM 81.12 system of 0.″197 makes the same longitude for the reference point of the 2.7 meter telescope at the McDonald Observatory as obtained by LLR.

If the rotation applied, the datum shift correction from the Tokyo Datum to the global geocentric coordinate system which is referred to the astronomical longitude system by LLR becomes

$$\Delta U = -146.0 \text{ m}$$
$$\Delta V = +506.7 \text{ m}$$
$$\Delta W = +681.2 \text{ m} .$$

If this correction is applied to the point of the origin of the Tokyo Datum, the new expression of the origin is as followings:

35° 39′ 29.″217N (latitude)
139 44 28 . 878E (longitude)

where A_e = 6378137.0 m, and $1/f$ = 298.257.

The precise comparison of these values with other coordinate systems, estimation of accuracies and more discussion will be made after processing much more data in the future.

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レーザー測距データによる衛星軌道と測地パラメーターの決定法およびその予備結果(要旨)

佐々木　稔

下里水路観測所に衛星レーザー測距装置を設置し，1982年3月から観測を続けている．得た測距データを用いて衛星の軌道と観測局の位置などの各種測地パラメーターを求めるための数値積分法による精密軌道決定プログラムを開発した．このプログラムは，衛星の軌道に影響を及ぼす，地球重力場，月・太陽・惑星による摂動力，光輻射圧，残留大気による抵抗力，地球潮汐および海洋潮汐の効果を含むものとなっている．衛星の軌道と測地パラメーターの決定法およびその計算式を示した．

次に，下里水路観測所および世界各地の観測局において得た測距値を，このプログラムで処理して同観測所の位置を，地球重心を原点とする世界測地系に基づいて予備的に求めた．このための世界測地系としては，LPM 81.12 システムを採用した．得た同観測所のレーザー測距装置の架台の高度・方位軸の交点の位置は，北緯 33° 34′ 39″.697，東経 135° 56′ 13″.156，基準楕円体（長半径6378 137.0m 扁平率1/298.257）からの高さは，100.66m である．日本測地系に基づく同地点の測地測量の結果との比較から，日本測地系で表わされた任意の地点の経緯度を，この LPM81.12 システムに変換するための座標変換量は，ΔU＝－142.5m，ΔV＝＋510.4m，ΔW＝＋681.2 m となった．この変換量を基に，日本測地原点のある地点の位置を LPM81.12システムで表わした場合の経緯度は，現行の日本測地系に採用されている値を基準として，緯度　＋11″.71，経度　－11″.83だけ異っており，これは，方位321°の方向に468m 移動した量に相当する．

一方，月レーザー測距が行なわれているマクドナルド天文台の口径2.7m の望遠鏡の基準点の経度について，月レーザー測距データの解析結果に基づく値と，この LPM81.12システムを東まわりに0″.197回転させた値とが一致すると言われる．この回転を全 LPM81.12システムに加えると，下里水路観測所の位置から求まる上記座標変換量は，ΔU＝－146.0m，ΔV＝＋506.7m，ΔW＝＋681.2m となり，この変換量を日本測地原点に適用した，月の経度システムに基づく日本測地原点の経緯度は，北緯　35° 39′ 29″.217，東経139° 44′ 28″.878と求まる．